Published by Institute of Physics Publishing for SISSA/ISAS
Received: *xxx, 2007*
Accepted: *xxx, 2007*

JHEP00(2007)000

Gravity Dual of Superconformal Anomaly

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Abstract: The supergravity dual of superconformal anomaly in a four-dimensional supersymmetric gauge theory is investigated. We consider a well-established dual correspondence between the $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ supersymmetric gauge theory with two flavors of matter fields in the bifundamental representation of gauge group and type IIB superstring in the space-time background furnished by the Klebanov-Strassler (K-S) solution. The D-brane configuration for these two dual theories consists of N $D3$ branes and M fractional $D3$ branes in the singular space-time composed of a direct product of M^4 and a six-dimensional conifold $\mathcal{C}_6$ with the base $T^{1,1}$. The superconformal anomaly originates from fractional branes frozen at the apex of $\mathcal{C}_6$. While on the gravity side, the fractional branes deform the $AdS_5\times T^{1,1}$ space-time background and partially break local supersymmetry of type IIB supergravity. We choose the K-S solution as the vacuum configuration for type IIB supergravity and observe the five-dimensional gauged supergravity produced from the spontaneous compactification on the deformed $T^{1,1}$. Consequently, we find that the deformation on $AdS_5\times T^{1,1}$ leads to the spontaneous breaking of supersymmetry in gauged AdS_5 supergravity and a super-Higgs mechanism arises. The graviton multiplet dual to the superconformal current of supersymmetric gauge theory becomes massive with the Goldstone multiplet relevant to the fluxes carried by fractional branes. We thus conclude that the super-Higgs mechanism in gauged supergravity is dual to the superconformal anomaly of supersymmetric gauge theory in terms of gauge/gravity correspondence.

Keywords: Gauge/gravity dual, fractional brane, superconformal anomaly, Klebanov-Strassler background, spontaneous compactification, super-Higgs mechanism.

Contents

1. Introduction

The AdS/CFT correspondence and its generalization, gauge/gravity duality, describe a physical equivalence between a supersymmetric gauge theory and a gravitational system [1, 2, 3, 4]. This amazing correspondence originates from open/closed string duality bridged

by D-brane, a solitonic object in string theory. A D-bane has two distinct features [5]: on one hand, in weakly coupled type-II superstring theory, it behaves as a dynamical and geometrical object with open strings ending on it. Thus a stack of coincided Dp-branes at low-energy will yield a $p+1$-dimensional supersymmetric gauge theory on the world-volume of Dp-branes. In principle, one can use various D-branes and NS solitonic branes as well as other geometric objects like orientifold planes to build any possible brane configurations and construct a anticipated gauge theory [6, 7]; On the other hand, a Dp-brane carries R-R charge and couples with $p+1$-rank antisymmetric field obtained from the R-R boundary condition in closed string theory. So it provides a source to the low-energy effective theory of strongly coupled type-II superstring theory — type-II supergravity. Therefore, a stack of Dp-branes can modify space-time background of string theory and emerge as a p-brane solution to type-II supergravity [8, 9]. This is the essence for gauge/gravity duality.

A physical phenomenon in one theory should have a physically equivalent description in the dual theory in terms of gauge/gravity duality. The quantum anomaly, a violation of classical symmetry by quantum correction, is a typical quantum phenomenon in field theory. The investigation on the dual description of anomaly can reveal some important features in gauge/gravity duality. Not long time ago, Klebanov, Ouyang and Witten investigated the gravity dual of $U(1)_R$ R-symmetry anomaly of $N=1$ cascading $SU(N+M)\times SU(N)$ gauge theory [10] in terms of its dual theory — type IIB supergravity in the background furnished by K-S solution [11]. The K-S solution is non-singular and originates from the brane configuration consisting of N bulk $D3$-branes and M fractional $D3$-branes fixed at the apex of conifold $\mathcal{C}_6$ in the target space-time $M_4\times\mathcal{C}^6$ [12]. If the fractional branes are absent in the brane configuration, the resultant space-time background is three-brane solution with near-horizon limit $AdS_5\times T^{1,1}$. The spontaneous compactification on $T^{1,1}$ of type IIB supergravity yields $\mathcal{N}=2$ $U(1)$ gauged AdS_5 supergravity. The UV limit of K-S solution can be considered a deformed $AdS_5\times T^{1,1}$. Klebanov *et al* showed that the $U(1)_R$ anomaly of $\mathcal{N}$=1 $SU(N+M)\times SU(N)$ supersymmetric gauge theory is dual to a spontaneous breaking of $U(1)$ gauge symmetry in gauged AdS_5 supergravity [13, 14] and the consequent Higgs mechanism.

However, in a classical scale invariant $\mathcal{N}=1$ supersymmetric gauge theory, the chiral R-symmetry current j^μ, the supersymmetry current s^μ and energy-momentum tensor $\theta^{\mu\nu}$, (j^μ,s^μ,$\theta^{\mu\nu}$), constitute a current supermultiplet [15, 16]. At quantum level the scale symmetry, chiral R-symmetry and conformal supersymmetry in an $\mathcal{N}<4$ supersymmetric gauge theory become anomalous due to the non-vanishing beta function of the theory. The divergence $\partial_\mu j^\mu$ of chiral R-symmetry current j^μ, the γ-trace $\gamma_\mu s^\mu$ of supersymmetry current s^μ and the trace $\theta^\mu_{\ \mu}$ of energy-momentum tensor $\theta_{\mu\nu}$ also constitute an anomaly supermultiplet with respect to the Poincaré supersymmetry [17]. Therefore, one may ask what the dual of the whole superconformal anomaly is. A natural guess is that it should correspond to the spontaneous breaking of local supersymmetry and the consequent super-Higgs mechanism in gauged AdS_5 supergravity. The aim of this paper is to show quantitatively how this dual description to the superconformal anomaly indeed stands there.

In Sect. 2, we first use the $\mathcal{N}=1$ supersymmetric Yang-Mills theory to illustrate the superconformal current and its anomaly supermultiplet. To observe the origin of the su-

perconformal anomaly in the brane configuration, we show how superconformal anomaly is reflected in the local part of gauge invariant quantum effective action. Further, we introduce the $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ supersymmetric gauge theory with two chiral matter superfields A_i, B_i $(i=1,2)$ in the bifundamental representation of gauge group and its superconformal anomaly. Sect. 3 displays the origin of superconformal anomaly in D-brane configuration. We review the $D3$/fractional $D3$ brane system in the singular space-time $M^4\times\mathcal{C}_6$. This brane configuration leads to $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ supersymmetric gauge theory. By expanding the Dirac-Born-Infeld (DBI) action and Wess-Zumino (WZ) term for a D-brane configuration and comparing with the quantum effective action of the supersymmetric gauge theory, we demonstrate that the superconformal anomaly originates from the fractional brane locating in the singularity of background space-time. We further introduce the K-S solution and its UV limit for later use. In Sect. 4, we analyze the manifestation of various symmetries on K-S solution. By comparing with three-brane solution without fractional branes, we show how the presence of fractional branes breaks some of local symmetries. We emphasize that it is the NS-NS and R-R two-form fields relevant to fractional branes that break $U(1)$ rotation- and scale transformation invariance in the transverse space of the above D-brane configuration. These two geometrical symmetries in the brane configuration are the global $U(1)_R$- and scale symmetry of the supersymmetric gauge theory living on the world-volume of $D3$-branes. Further, the Killing spinor equation in the K-S solution background tells how half of supersymmetries disappear due to the modification on $AdS_5\times T^{1,1}$ by fractional branes. In Sect. 5 we choose the K-S solution as a vacuum configuration for type IIB supergravity and observe the dynamical phenomenon. The spontaneous compactification on the deformed $T^{1,1}$ occurs and a five-dimensional gauged supergravity arises. In contrast to the case in the $AdS_5\times T^{1,1}$ background, which gives the $\mathcal{N}=2$ $U(1)$ gauged AdS_5 supergravity coupled with $SU(2)\times SU(2)$ Yang-Mills vector multiplets as well as some Betti scalar, vector and tensor multiplets, we find that the local supersymmetry, diffeomorphism symmetry and $U(1)$ gauge symmetry partially break in the gauged AdS_5 supergravity since the deformed $AdS_5\times T^{1,1}$ possesses less isometric symmetries and preserves less supersymmetries. We then go to reveal the super-Higgs mechanism corresponding to the spontaneous breaking of the above local symmetries. We exhibit the process how the graviton, $U(1)$ gauge field and gravitino in AdS_5 space become massive through "eating" a Goldstone supermutiplet originating from the NS-NS- and R-R fluxes carried by fractional branes. Since the deformation on $AdS_5\times T^{1,1}$ is produced by fractional branes in the brane configuration and meanwhile they are the origin for superconformal anomaly on the field theory side, we thus claim that the gravity dual of superconformal anomaly is the spontaneous breaking of local supersymmetry and the consequent super-Higgs effect in gauged AdS_5 supergravity. Sect. 6 is a brief summary.

2. Superconformal anomaly multiplet of $\mathcal{N}=1$ four-dimensional supersymmetric gauge theory

2.1 $\mathcal{N}=1$ supersymmetric $SU(N)$ Yang-Mills theory as an illustration

In a an $\mathcal{N}=1$ supersymmetric field theory, the Poincaré supersymmetry combines the

energy-momentum tensor $\theta^{\mu\nu}$, the supersymmetry current s^μ and the axial vector (or equivalently chiral) R-symmetry current j^μ into a supercurrent multiplet. If a field model has scale symmetry, these currents are not only conserved at classical level,

$$\partial_\mu\theta^{\mu\nu}=\partial_\mu s^\mu=\partial_\mu j^\mu=0, \tag{2.1}$$

but also satisfy algebraic relations, $\theta^\mu{}_\mu=\gamma_\mu s^\mu=0$. Consequently, three more conserved currents can be constructed,

$$d^\mu\equiv x_\nu\theta^{\nu\mu}, \quad k_{\mu\nu}\equiv 2x_\nu x^\rho\theta_{\rho\mu}-x^2\theta_{\mu\nu}, \quad l_\mu\equiv ix^\nu\gamma_\nu s_\mu. \tag{2.2}$$

The charges D, K_μ and S_α of these three new currents are the generators for dilatation, special conformal symmetry- and conformal supersymmetry transformations. They constitute an $\mathcal{N}=1$ superconformal algebra $SU(2,2|1)$ together with Lorentz generators $M_{\mu\nu}$, translation generator P_μ and supercharge Q_α. Consequently, the Poincaré supersymmetry promotes to a larger superconformal symmetry.

However, the superconformal symmetry becomes anomalous at quantum level. For an $\mathcal{N}<4$ supersymmetric gauge theory, due to the renormalization effect, an energy scale is be generated dynamically. Consequently, the scale anomaly and the subsequent superconformal anomaly multiplet required by the Poincaré supersymmetry must arise. In the case that all of them, the trace $\theta^\mu{}_\mu$ of energy-momentum tensor $\theta_{\mu\nu}$, the γ-trace $\gamma^\mu s_\mu$ of supersymmetry current s_μ and the divergence $\partial_\mu j^\mu$ of the chiral R-symmetry current j_μ, get contribution from quantum correction, $(\partial_\mu j^\mu,\gamma^\mu s_\mu,\theta^\mu{}_\mu)$ will form a chiral supermultiplet with $\partial_\mu j^\mu$ playing the role of the lowest component of a chiral superfield [15, 16].

In the following we take $\mathcal{N}=1$ supersymmetric $SU(N)$ Yang-Mills theory as an illustrating example. Its field content consists of a vector field A_μ, a Majorana spinor λ and an auxiliary field D in the adjoint representation of $SU(N)$. The classical action of the theory is

$$S=\int d^4x\left[-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a}+\frac{1}{2}i\overline{\lambda}^a\gamma^\mu(\nabla_\mu\lambda)^a+\frac{1}{2}\left(D^a\right)^2\right]. \tag{2.3}$$

The superconformal anomaly multiplet at quantum level is [17]

$$\begin{aligned}
\partial_\mu j^\mu &= 2\frac{\beta(g)}{g}\left[-\frac{1}{6}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}+\frac{1}{3}\partial^\mu\left(\overline{\lambda}^a\gamma_\mu\gamma_5\lambda^a\right)\right],\\
\gamma^\mu s_\mu &= 2\frac{\beta(g)}{g}\left(-\sigma^{\mu\nu}G^a_{\mu\nu}+\gamma_5 D^a\right)\lambda^a,\\
\theta^\mu{}_\mu &= 2\frac{\beta(g)}{g}\left[-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a}+\frac{1}{2}i\overline{\lambda}^a\gamma^\mu(\nabla_\mu\lambda)^a+\frac{1}{2}\left(D^a\right)^2\right],
\end{aligned} \tag{2.4}$$

In above equation, $\beta(g)$ is the β-function for the gauge coupling of the $\mathcal{N}=1$ $SU(N)$ super-Yang-Mills theory. At one-loop order, $\beta(g)=-3Ng^3/(16\pi^2)$.

Eq. (2.4) shows that the anomaly coefficients are proportional to the beta function of $\mathcal{N}=1$ supersymmetric Yang-Mills theory [17, 18, 19]. This is a universal feature for any $\mathcal{N}<4$ supersymmetric field theory. The origins of scale anomaly $\theta^\mu{}_\mu$ and chiral anomaly

$\partial_\mu j^\mu$ are the same as in usual gauge field theories: the UV divergence in perturbation theory requires a renormalization procedure to make the theory well defined in which an energy scale parameter must be introduced, hence the scale anomaly arises. The origin of chiral $U_R(1)$ anomaly is similar to that of the axial $U_A(1)$ anomaly in QCD. It comes from the contradiction of $U_R(1)$ symmetry with the global vector $U(1)$ rotational symmetry among gauginoes at quantum level. The $U_R(1)$ symmetry thus becomes anomalous when the vector $U(1)$ symmetry is required to stand. The outcome of the γ-trace anomaly lies in the incompatibility between the conservation and the vanishing γ-trace of supersymmetry current s^μ at quantum level [19]. Usually, from the physical consideration that the supersymmetry current should be conserved, the γ-trace anomaly thus takes place.

The superconformal anomaly manifest itself in the quantum effective action of the theory. We first re-scale the fields $(A_\mu, \lambda, D) \to (A_\mu/g, \lambda/g, D/g)$ and consider the strong CP violation term. Then the classical action of $\mathcal{N}=1$ supersymmetric $SU(N)$ Yang-Mills theory can be re-written as

$$S_{\rm cl} = \frac{1}{g^2}\int d^4x \left[-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \frac{1}{2}i\overline{\lambda}^a\gamma^\mu(\nabla_\mu\lambda)^a + \frac{1}{2}\left(D^a\right)^2\right] + \frac{\theta^{(\rm CP)}}{32\pi^2}\int d^4x G^a_{\mu\nu}\widetilde{G}^{\mu\nu a}. \tag{2.5}$$

The local part of the gauge invariant quantum effective action takes the form,

$$\begin{aligned}\Gamma_{\rm eff} &= \frac{1}{g^2_{\rm eff}}\int d^4x \left[-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \frac{1}{2}i\overline{\lambda}^a\gamma^\mu(\nabla_\mu\lambda)^a + \frac{1}{2}\left(D^a\right)^2\right]\\ &\quad + \frac{\theta^{(\rm CP)}_{\rm eff}}{32\pi^2}\int d^4x G^a_{\mu\nu}\widetilde{G}^{\mu\nu a},\end{aligned} \tag{2.6}$$

where all the fields are renormalized quantities. The scale- and chiral anomalies are reflected in the running of gauge coupling and the shift of θ-angle due to the non-vanishing $\beta(g)$ [20],

$$\begin{aligned}\frac{1}{g^2_{\rm eff}(q^2)} &= \frac{1}{g^2(q_0^2)} + \frac{3N}{8\pi^2}\ln\frac{q_0}{q} \equiv \frac{3N}{8\pi^2}\ln\frac{q}{\Lambda},\\ \theta^{(\rm CP)}_{\rm eff} &= \theta^{(\rm CP)} + 3N.\end{aligned} \tag{2.7}$$

To recognize the γ-trace anomaly of supersymmetry current in the quantum effective action, we must resort to its superfield form. First, the superfield form of the classical Lagrangian (2.5) is

$$\mathcal{L} = \frac{1}{32\pi}\int d^2\theta\, {\rm Im}\left[\tau {\rm Tr}\left(W^\alpha W_\alpha\right)\right] + {\rm h.c.}. \tag{2.8}$$

In above equation, W_α is the field strength of superspace gauge connection and in the Wess-Zumino gauge, $W_\alpha = \lambda_\alpha - \frac{i}{2}\left(\sigma^{\mu\nu}\right)_{\alpha\beta}\theta^\beta F_{\mu\nu} + i\theta_\alpha D - i\theta^2\left(\sigma^\mu\right)_{\alpha\dot{\alpha}}\nabla\overline{\lambda}^{\dot{\alpha}}$. The parameter τ is a complex combination of gauge coupling and θ-angle,

$$\tau = \frac{\theta^{(\rm CP)}}{2\pi} + i\frac{4\pi}{g^2}. \tag{2.9}$$

To show how the conformal supersymmetry anomaly $\gamma_\mu s^\mu$ is reflected in the quantum effective action, we define that τ is the lowest component of a certain constant chiral superfield, $\Sigma \equiv \tau + \theta\chi + \theta^2 d$. Further, we require that classically $\chi_{\rm cl} = d_{\rm cl} = 0$ and assume only at quantum level χ and d get non-vanishing expectation values,

$$\Sigma_{\rm eff} \equiv \tau_{\rm eff} + \theta\chi_{\rm eff} + \theta^2 d_{\rm eff}, \tag{2.10}$$

where $\theta_{\rm eff}^{(\rm CP)}$, $g_{\rm eff}^2$ and hence $\tau_{\rm eff}$ are given in (2.7). In the Wess-Zumino gauge, the variation of the local quantum effective action in superspace produced from the shift of $\Sigma_{\rm eff}$ reads,

$$\begin{aligned}
\delta\Gamma_{\rm eff} &= \frac{1}{32\pi}\int d^2\theta\, \mathrm{Im}\left[\delta\Sigma_{\rm eff}\mathrm{Tr}\left(W^\alpha W_\alpha\right)\right] + \text{h.c.} \\
&= \frac{1}{32\pi}\int d^2\theta\, \mathrm{Im}\left\{\left(\delta\tau_{\rm eff} + \theta\delta\chi_{\rm eff} + \theta^2\delta d_{\rm eff}\right)\left[\lambda\lambda + i\theta\left(2\lambda D + \sigma^{\mu\nu}\lambda G_{\mu\nu}\right)\right.\right. \\
&\quad \left.\left. +\theta^2\left(-\frac{1}{2}G_{\mu\nu}G^{\mu\nu} + \frac{i}{2}G_{\mu\nu}\widetilde{G}^{\mu\nu} - 2i\lambda\sigma^\mu\nabla_\mu\overline{\lambda} - D^2\right)\right]\right\} + \text{h.c.}.
\end{aligned} \tag{2.11}$$

A comparison between Eq. (2.11) and the superconformal anomaly equation (2.4) reveals that the γ-trace anomaly of supersymmetry current is reflected in the shift of the fermionic parameter χ, which is exactly like the trace- and chiral anomalies are represented by the running of the gauge couplings and the shift of θ-angle. Note that this is not the whole advantage of promoting τ parameter to a chiral superfield. Later we shall later that from the viewpoint of brane dynamics, the running of gauge coupling and the shift of θ angle originate from fractional branes locating at the space-time singularity and further can get down to the Goldstone fields. Therefore, introducing an artificial superpartner for the gauge coupling and θ-angle is helpful for identifying the Goldstone supermultiplet for the super-Higgs mechanism on the gauged AdS_5 supergravity side.

2.2 $\mathcal{N} = 1$ supersymmetric $SU(N+M) \times SU(N)$ gauge theory with two flavors in bifundamental representations $(N+M, \overline{N})$ and $(\overline{N+M}, N)$

The reason why we specialize to $\mathcal{N} = 1$ supersymmetric $SU(N+M)\times SU(N)$ gauge theory with two flavors in the bifundmental representations $(N+M, \overline{N})$ and $(\overline{N+M}, N)$ of gauge group is that its supergravity dual is fully understood, which is the type IIB superstring in the space-time background found by Klebanov and Strassler [11]. We first write down the classical action of this supersymmetric gauge theory in superspace,

$$\begin{aligned}
S = \int d^4x \left\{\frac{1}{32\pi}\int d^2\theta \sum_{i=1}^{2}\mathrm{Im}\left[\tau_{(i)}\mathrm{Tr}\left(W^{(i)\alpha}W_\alpha^{(i)}\right)\right] + \text{h.c.}\right. \\
\left. + \int d^2\theta d^2\overline{\theta}\sum_{j=1}^{2}\left(\overline{A}^j e^{\left[g_1 V^{(1)}(N+M) + g_2 V_2(\overline{N})\right]}A_j + \overline{B}_j e^{\left[g_1 V^{(1)}(\overline{N+M}) + g_2 V_2(N)\right]}B^j\right)\right\}
\end{aligned} \tag{2.12}$$

where the two flavors A_j and B^j are chiral superfields in the bifundamental representations $(N+M, \overline{N})$ and $(\overline{N+M}, N)$, respectively,

$$\left(A_j\right)^m_{\ r} = \left(\phi_{Aj}\right)^m_{\ r} + \theta\left(\psi_{Aj}\right)^m_{\ r} + \theta^2\left(F_{Aj}\right)^m_{\ r},$$

$$
\begin{aligned}
\left(B^j\right)^r{}_m &= \left(\phi_B^j\right)^r{}_m + \theta\left(\psi_B^j\right)^r{}_m + \theta^2\left(F_B^j\right)^r{}_m, \\
& j=1,2, \quad r=1,\cdots,N+M, \quad m=1,\cdots,N.
\end{aligned} \tag{2.13}
$$

This model also admits a quartic superpotential, $W = \lambda\epsilon^{ik}\epsilon^{jl}\mathrm{Tr}\left(A_iB_jA_kB_l\right)$. Since the superpotential should have R-charge 2, this quartic superpotential normalizes the R-charge of the chiral superfields to be 1/2. Consequently, their fermionic components have R-charge $-1/2$. This model is a vector supersymmetric gauge theory. If we consider A_j as quark chiral superfields, then B^j are the corresponding anti-quark chiral superfields. In the Wess-Zumino gauge, the two-component form of above action is

$$
\begin{aligned}
\mathcal{L} &= \sum_{i=1}^{2}\frac{1}{g_{(i)}^2}\left[-\frac{1}{4}G^{(i)a_i\mu\nu}G^{(i)a_i}_{\mu\nu} + i\lambda^{(i)a_i}\sigma^\mu\left(\nabla_\mu\overline{\lambda}^{(i)}\right)^{a_i} + \frac{1}{2}D^{(i)a_i}D^{(i)a_i}\right] + \frac{\theta_{(i)}^{(\mathrm{CP})}}{32\pi^2}G^{(i)a_i\mu\nu}\widetilde{G}^{(i)a_i}_{\mu\nu} \\
&+ \left(D^\mu\phi_A\right)^{\dagger j}D_\mu\phi_{Aj} + \widetilde{D}^\mu\phi_B^j\left(\widetilde{D}_\mu\phi_B\right)^\dagger_j + i\overline{\psi}^j_A\overline{\sigma}^\mu D_\mu\psi_{Aj} + i\psi_B^j\sigma^\mu\widetilde{D}_\mu\overline{\psi}_{Bj} \\
&+ \sqrt{2}i\sum_{i=1}^{2}(-1)^{i+1}g_{(i)}\left[\phi_A^{\dagger j}T^{a_i}\lambda^{a_i}_{(i)}\psi_{Aj} - \overline{\lambda}^{a_i}_{(i)}\overline{\psi}^j_AT^{a_i}\phi_{Aj} - \psi_B^j\lambda^{a_i}_{(i)}T^{a_i}\phi^\dagger_{Bj} + \phi_B^jT^{a_i}\overline{\psi}_{Bj}\overline{\lambda}^{a_i}_{(i)}\right] \\
&+ \sum_{i=1}^{2}g_{(i)}(-1)^{i+1}D^{a_i}_{(i)}\left[\phi_A^{\dagger j}T^{a_i}\phi_{Aj} - \phi_B^jT^{a_i}\phi^\dagger_{Bj}\right] \\
&+ F_A^{\dagger j}F_{Aj} + F_B^jF^\dagger_{Bj} + \left[\left(F_{Aj}\frac{\partial W}{\partial\phi_{Aj}} + F_B^j\frac{\partial W}{\partial\phi_B^j}\right)\right. \\
&+ \left.\frac{1}{2}\left(\frac{\partial^2W}{\partial\phi_{Ai}\partial\phi_{Aj}}\psi_{Ai}\psi_{Aj} + \frac{\partial^2W}{\partial\phi_B^i\partial\phi_B^j}\psi_B^i\psi_B^j + 2\frac{\partial^2W}{\partial\phi_{Ai}\partial\phi_B^j}\psi_{Ai}\psi_B^j\right) + \mathrm{h.c.}\right],
\end{aligned} \tag{2.14}
$$

where

$$
\begin{aligned}
\left(\nabla_\mu\overline{\lambda}_{(i)}\right)^{a_i} &= \partial_\mu\overline{\lambda}^{a_i}_{(i)} + gf^{a_ib_ic_i}A^{b_i}_{(i)\mu}\overline{\lambda}^{c_i}_{(i)}, \\
\left(D_\mu\psi_{Aj}\right)^m{}_r &= \partial_\mu\psi^m_{Ajr} + ig_1A^{a_1}_{(1)\mu}\left(T^{a_1}\right)_r{}^s\psi^m_{Ajs} - ig_2\psi^n_{Ajr}\left(T^{a_2}\right)_n{}^mA^{a_2}_{(2)\mu}, \\
\left(\widetilde{D}_\mu\overline{\psi}_{Bj}\right)^m{}_r &= \partial_\mu\overline{\psi}^m_{Bjr} + ig_1\overline{\psi}^m_{Bjs}\left(T^{a_1}\right)^s{}_rA^{a_1}_{(1)\mu} - ig_2A^{a_2}_{(2)\mu}\left(T^{a_2}\right)^m{}_n\overline{\psi}^n_{Bjr}, \\
\left(D_\mu\phi_{Aj}\right)^m{}_r &= \partial_\mu\phi^m_{Ajr} + ig_1A^{a_1}_{(1)\mu}\left(T^{a_1}\right)_r{}^s\phi^m_{As} - ig_2\phi_{Ajn}\left(T^{a_2}\right)^n{}_mA^{a_2}_{(2)\mu}, \\
\left(D_\mu\phi_A^j\right)^{*r}{}_m &= \partial_\mu\phi^{*jr}_{Am} - ig_1\phi^{*js}_{Am}\left(T^{a_1}\right)_s{}^rA^{a_1}_{(1)\mu} + ig_2A^{a_2}_{(2)\mu}\left(T^{a_2}\right)_m{}^n\phi^{*jr}_{An}, \\
\left(\widetilde{D}_\mu\phi_B^j\right)^r{}_m &= \partial_\mu\widetilde{\phi}^{jr}_{Bm} - ig_1\phi^s_{Bm}\left(T^{a_1}\right)_s{}^rA^{a_1}_{(1)\mu} + ig_2A^{a_2}_{(2)\mu}\left(T^{a_2}\right)_m{}^n\phi^{jr}_{Bn}, \\
\left(\widetilde{D}_\mu\phi_{Bj}\right)^{*m}{}_r &= \partial_\mu\phi^{*m}_{Bjr} + ig_1A^{a_1}_{(1)\mu}\left(T^{a_1}\right)_r{}^s\phi^{*m}_{Bjs} - ig_2\phi^{*n}_{Bjs}\left(T^{a_2}\right)_n{}^mA^{a_2}_{(2)\mu}.
\end{aligned} \tag{2.15}
$$

The theory has global symmetry $SU_L(2)\times SU_R(2)\times U_B(1)\times U_A(1)$ at classical level and the $U_A(1)$ symmetry becomes anomalous at quantum level. In addition, the theory has a chiral $U_R(1)$ R-symmetry which rotates the left- and right-handed components of $\mathcal{N}=1$ supercharges. It is this chiral $U_R(1)$ symmetry anomaly that enters the superconformal anomaly multiplet.

Let us list the supercurrent- and superconformal anomaly multiplets of this field model to facilitate the analysis on superconformal anomaly. The supercurrent multiplet (in four-component form) is

$$\begin{aligned}
\theta_{\mu\nu} =& \sum_{i=1}^{2}\left\{-G^{(i)a_i}_{\mu\rho}G^{(i)a_i\rho}_{\nu}+\frac{1}{2}i\left[\overline{\lambda}^{a_i}(\gamma_\mu\nabla_\nu+\gamma_\nu\nabla_\mu)\lambda^{a_i}-(\nabla_\mu\overline{\lambda}^{a_i}\gamma_\nu+\nabla_\nu\overline{\lambda}^{a_i}\gamma_\mu)\lambda^{a_i}\right]\right.\\
&\left.+\frac{1}{4}g_{\mu\nu}G^{(i)a_i}_{\lambda\rho}G^{(i)a_i\lambda\rho}-\frac{1}{4}ig_{\mu\nu}\left[\overline{\lambda}^{a_i}\gamma^\rho\left(\nabla_\rho\lambda^{a_i}\right)-\left(\nabla_\rho\overline{\lambda}^{a_i}\right)\gamma^\rho\lambda^{a_i}\right]\right\}\\
&+\left[\left(D_\mu\phi_A\right)^{\dagger j}D_\nu\phi_{Aj}+\left(D_\nu\phi_A\right)^{\dagger j}D_\mu\phi_{Aj}\right]+\left[\left(\widetilde{D}_\mu\phi_B\right)^\dagger_j\widetilde{D}_\nu\phi^j_B+\left(\widetilde{D}_\nu\phi_B\right)^\dagger_j\widetilde{D}_\mu\phi^j_B\right]\\
&-\frac{1}{3}\left(\partial_\mu\partial_\nu-g_{\mu\nu}\partial^2\right)\left(\phi^{\dagger j}_A\phi_{Aj}\right)-\frac{1}{3}\left(\partial_\mu\partial_\nu-g_{\mu\nu}\partial^2\right)\left(\phi^\dagger_{Bj}\phi^j_B\right)\\
&+i\overline{\psi}^j\left(\gamma_\mu D_\nu+\gamma_\nu D_\mu\right)\psi_j-g_{\mu\nu}\left\{i\overline{\psi}^j\gamma^\lambda D_\lambda\psi_j+\left(D^\lambda\phi_A\right)^{\dagger j}D_\mu\phi_{Aj}+\widetilde{D}^\lambda\phi^j_B\left(\widetilde{D}_\lambda\phi_B\right)^\dagger_j\right.\\
&+\frac{i}{\sqrt{2}}\sum_{i=1}^{2}(-1)^{i+1}g_{(i)}\left[\phi^{\dagger j}_AT^{a_i}\overline{\lambda}^{a_i}(1-\gamma_5)\psi_j-\overline{\psi}^j(1+\gamma_5)\lambda^{a_i}T^{a_i}\phi_{Aj}\right.\\
&\left.-\overline{\psi}^j(1-\gamma_5)\lambda^{a_i}T^{a_i}\phi^\dagger_{Bj}+\phi^j_BT^{a_i}\overline{\lambda}^{a_i}(1+\gamma_5)\psi_j\right]-\frac{1}{2}\sum_{i=1}^{2}g^2_{(i)}\left(\phi^{\dagger j}_AT^{a_i}\phi_{Aj}-\phi^j_BT^{a_i}\phi^\dagger_{Bj}\right)^2\\
&\left.+\text{superpotential terms}\right\}\\
s_\mu =& -\sum_{i=1}^{2}\sigma_{\nu\rho}\gamma_\mu\lambda^{a_i}G^{(i)a_i\nu\rho}+\frac{1}{2}\left(D^\nu\phi_A\right)^{\dagger j}\gamma_\nu(1+\gamma_5)\gamma_\mu\psi_j-\frac{1}{2}\widetilde{D}^\nu\phi^j_B\gamma_\nu(1-\gamma_5)\gamma_\mu\psi_j\\
&+\frac{1}{2}\gamma_\nu(1-\gamma_5)\gamma_\mu\left(C\overline{\psi}^{Tj}\right)D_\nu\phi_{Aj}-\frac{1}{2}\gamma_\nu(1-\gamma_5)\gamma_\mu\left(C\overline{\psi}^{Tj}\right)\left(\widetilde{D}_\nu\phi_B\right)^\dagger\\
&-\frac{1}{3}i\sigma_{\mu\nu}\partial^\nu\left[\phi_{Aj}(1+\gamma_5)\left(C\overline{\psi}^{Tj}\right)+\phi^{j\dagger}_A(1-\gamma_5)\psi_j\right]\\
&+\frac{1}{3}i\sigma_{\mu\nu}\partial^\nu\left[\phi^\dagger_{Bj}(1-\gamma_5)\left(C\overline{\psi}^{Tj}\right)+\phi^j_B(1+\gamma_5)\psi_j\right]\\
&+2\sqrt{2}\sum_{i=1}^{2}(-1)^{i+1}g_{(i)}\gamma_5\gamma_\mu\lambda^{a_i}\left[\phi^{j\dagger}_AT^{a_i}\phi_{Aj}-\phi_{Bj}T^a\phi^{\dagger j}_B\right]+\text{superpotential terms;}\\
j_\mu =& \frac{1}{2}\sum_{i=1}^{2}\overline{\lambda}^{a_i}\gamma_\mu\gamma_5\lambda^{a_i}-\frac{1}{3}\overline{\psi}^j\gamma_\mu\gamma_5\psi_j-\frac{2i}{3}\left[\phi^{\dagger j}_AD_\mu\phi_{Aj}-\left(D_\mu\phi_A\right)^{\dagger j}\phi_{Aj}\right]\\
&-\frac{2i}{3}\left[\phi^j_B\left(\widetilde{D}_\mu\phi_{Bj}\right)^\dagger-\left(\widetilde{D}_\mu\phi_B\right)^j\phi^\dagger_{Aj}\right]+\text{superpotential terms.}
\end{aligned}\tag{2.16}$$

In above equation, λ^a and ψ are Majorana and Dirac spinors, respectively,

$$\lambda^a=\begin{pmatrix}\lambda^a_\alpha\\ \overline{\lambda}^{a\dot{\alpha}}\end{pmatrix},\quad \psi_j=\begin{pmatrix}\psi_{Aj\alpha}\\ \overline{\psi}^{\dot{\alpha}}_{Bj}\end{pmatrix},\quad \left(C\overline{\psi}^T\right)^j=\begin{pmatrix}\psi^j_{B\alpha}\\ \overline{\psi}^{j\dot{\alpha}}_A\end{pmatrix}. \tag{2.17}$$

Classically the above conservative currents satisfy $\theta^\mu{}_\mu=\gamma^\mu s_\mu=0$ up to the classical superpotential terms. At quantum level the superconformal anomaly arises due to the non-vanishing β-functions of two gauge couplings. We first analyze the chiral $U_R(1)$ symmetry

anomaly. Eq. (2.17) shows that the chiral current j^μ is composed of gluinoes λ^{a_i}, $(\psi_{Aj})^m_{\ r}$ and $\left(\psi^j_B\right)^r_{\ m}$. Relative to the first gauge group $SU(N+M)$, there are $2N$ flavor matters (counting m index in $(\psi_{Aj})^m_{\ r}$ and $\left(\psi^j_B\right)^r_{\ m}$), which contribute $2N\times(-1/2)=-N$ to the anomaly coefficient. The gluino λ^{a_1} is in the adjoint representation of $SU(N+M)$ and hence makes the contribution $C_2[SU(N+M)]=N+M$ to the anomaly coefficient. So for the first gauge field background, the chiral anomaly coefficient is $N+M-N=M$. A similar analysis for the second gauge group $SU(N)$ gives the chiral anomaly coefficient $-M$. Therefore, we obtain the chiral $U_R(1)$ anomaly,

$$\partial_\mu j^\mu = \frac{M}{16\pi^2}\left(g_1^2 G^{a_1}_{\mu\nu}\widetilde{G}^{a_1\mu\nu} - g_2^2 G^{a_2}_{\mu\nu}\widetilde{G}^{a_2\mu\nu}\right) + \text{classical superpotential part.} \tag{2.18}$$

To observe how the scale anomaly arises, we first consider the $SU(N)\times SU(N)$ gauge theory with chiral superfields A_j in $(N,\overline{N})$ and B^j in $(\overline{N},N)$ representations. This theory is a superconformal field theory in the sense that it has IR fixed-points. From the NSVZ β-functions for those two gauge couplings,

$$\begin{aligned}\beta(g_1^2) &= -\frac{g_1^3}{16\pi^2}\left[3N-2N(1-\gamma(g))\right],\\ \beta(g_2^2) &= \frac{g_2^3}{16\pi^2}\left[3N-2N(1-\gamma(g))\right],\end{aligned} \tag{2.19}$$

we can see that the zero-points of β-functions arise at the anomalous dimension $\gamma(g)=-1/2$. However, for the $SU(N+M)\times SU(N)$ gauge theory with chiral superfields A_j in $(N+M,\overline{N})$ and B^j in $(N,\overline{N+M})$ representations, the IR fixed points are removed since now the corresponding β-functions become

$$\beta(g_1^2) = -\frac{3Mg_1^3}{16\pi^2}, \qquad \beta(g_2^2) = \frac{3Mg_2^3}{16\pi^2}. \tag{2.20}$$

So the superconformal anomaly should arise near the IR fixed points of the $M=0$ case . We have the scale anomaly similar to that listed in Eq. (2.4),

$$\begin{aligned}\theta^\mu_{\ \mu} &= \frac{3M}{8\pi^2}\sum_{i=1}^2(-1)^{i+1}g_i^2\left[-\frac{1}{4}G^{(i)a_i\mu\nu}G^{(i)a_i}_{\mu\nu} + \frac{1}{2}i\overline{\lambda}^{a_i}\gamma^\mu\left(\nabla_\mu\lambda\right)^{a_i}\right]\\ &\quad+\text{classical superpotential part.}\end{aligned} \tag{2.21}$$

Further, using the beta functions (2.20) near the IR-fixed points in the $M=0$ case, we easily find the γ-trace anomaly of supersymmetry current,

$$\gamma^\mu s_\mu = \frac{3M}{8\pi^2}\sum_{i=1}^2(-1)^{i+1}g_i^2\left(-\sigma^{\mu\nu}G^{a_i}_{\mu\nu}+\gamma_5 D^{a_i}\right)\lambda^{a_i}. \tag{2.22}$$

Let us turn to the manifestation of above superconformal anomaly in the quantum effective action. We consider the local part of the gauge invariant quantum effective action composed only of gauge fields,

$$\Gamma_{\text{eff}} = \frac{1}{32\pi}\int d^2\theta\sum_{i=1}^2\text{Im}\left[\Sigma_{(i)\,\text{eff}}\text{Tr}\left(W^{(i)\alpha}W^{(i)}_\alpha\right)\right] + \text{h.c.}, \tag{2.23}$$

where $\Sigma_{(i)\,\mathrm{eff}}$ is given in (2.10). A similar discussion as the case of $\mathcal{N}=1$ supersymmetric gauge theory shows that the superconformal anomaly manifest itself as the running of gauge couplings g_i and the shifts of both CP-angles θ_i^{CP} and the superpartner $\chi_{(i)}$ in the above quantum effective action,

$$\begin{aligned} \frac{1}{g^2_{(i)\,\mathrm{eff}}(q^2)} &= \frac{1}{g^2_{(i)}(q_0^2)} + (-1)^{i+1}\frac{3M}{8\pi^2}\ln\frac{q_0}{q}, \\ \theta^{(\mathrm{CP})}_{(i)\,\mathrm{eff}} &= \theta^{(\mathrm{CP})}_{(i)} + (-1)^{i+1}M, \\ \chi_{(i)\,\mathrm{eff}} &= 0 + (-1)^{i+1}M. \end{aligned} \tag{2.24}$$

This ends our discussion on the superconformal anomaly in the $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ supersymmetric gauge theory.

3. Superconformal Anomaly from Brane Dynamics

In this section we reveal the origin of superconformal anomaly in the brane configuration. This will pave the way for us to investigate the supergravity dual of superconformal anomaly of supersymmetric gauge theory.

3.1 Brane configuration for $\mathcal{N}=1$ supersymmetric $SU(N+M)\times SU(N)$ gauge theory and K-S solution

The brane configuration for the $\mathcal{N}=1$ supersymmetric $SU(N+M)\times SU(N)$ gauge theory has been constructed as the following [12]. One starts from the bulk space-time composed of the direct product of a four-dimensional Minkowski space M^4 and a six-dimensional conifold $\mathcal{C}_6$ whose base is $T^{1,1}=[SU(2)\times SU(2)]/U(1)$. The isometry group of $T^{1,1}$ is $SU(2)\times SU(2)\times U(1)$. Such a target space-time allows not only a stack of N $D3$-branes moving freely in the transverse space, but also a stack of M fractional $D3$-branes fixed at the singularity, the apex of $\mathcal{C}_6$. All these $D3$-branes extend out in M^4. Topologically $T^{1,1}\sim S^2\times S^3$, and the fractional $D3$-branes can be considered as $D5$-branes wrapped around two-cycle S^2.

The four-dimensional $\mathcal{N}=1$ supersymmetric gauge theory produced from above brane configuration can be found by observing the massless spectrum of open string with 4 Neumann and 6 Dirichlet boundary conditions in $M^4\times\mathcal{C}_6$ [12]. In the case with no fractional $D3$ branes, it turned out that the resultant field theory on the world volume of $D3$-brane is the $\mathcal{N}=1$ $SU(N)\times SU(N)$ super-Yang-Mills theory coupled with four chiral matter superfields, two of which $(A_i)^m{}_r$ transform as $(N,\overline{N})$ and the other two $\left(B^j\right)^r{}_m$ as $(\overline{N},N)$ under the product gauge group $SU(N)\times SU(N)$, $i=1,2$, $r=1,\cdots,N$, $m=1,\cdots,\overline{N}$. The global symmetry group $SU_L(2)\times SU_R(2)\times U_R(1)$ of the gauge theory comes from the isometry group of $T^{1,1}$. When M fractional branes are added at the apex of $\mathcal{C}_6$, the superconformal symmetry of this field model at the IR fixed points disappears and the resultant field theory is $SU(N+M)\times SU(N)$ super-Yang-Mills theory coupled with chiral superfields $(A_i)^m{}_r$ and $\left(B^j\right)^r{}_m$ in the bifundmental representations $(N+M,\overline{N})$ and $(\overline{N+M},N)$ of the gauge group.

On the gravity side, when no fractional $D3$-branes are present, the three-brane solution to type IIB supergravity corresponding to such a brane configuration takes the following form [12],

$$
\begin{aligned}
ds^2 &= H^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)\left(dr^2 + r^2 ds^2_{T^{1,1}}\right), \\
F_{(5)} &= \mathcal{F}_{(5)} + \widetilde{\mathcal{F}}_{(5)}, \\
H(r) &= 1 + \frac{L^4}{r^4}, \quad L^4 = \frac{27\pi g_s N(\alpha')^2}{4}, \\
ds^2_{T^{1,1}} &= \frac{1}{9}\left(2d\beta + \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right) \\
&= \frac{1}{9}(\sigma^{\widetilde{3}} + \sigma^{\widehat{3}})^2 + \frac{1}{6}\sum_{\widetilde{1}=1}^{2}(\sigma^{\widetilde{i}})^2 + \frac{1}{6}\sum_{\widehat{i}=1}^{2}(\sigma^{\widehat{i}})^2 = \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{m=1}^{4}(g^m)^2, \\
\mathcal{F}_{(5)} &= \frac{1}{2}\pi\alpha'^2 N \sin\theta_1 \sin\theta_2 d\beta \wedge d\theta_1 \wedge d\theta_2 \wedge d\phi_1 \wedge d\phi_2,
\end{aligned}
\tag{3.1}
$$

In above solution, g^m are one-form bases on $T^{1,1}$,

$$
\begin{aligned}
g^1 &\equiv \frac{\sigma^{\widetilde{1}} - \sigma^{\widehat{1}}}{\sqrt{2}}, \quad g^2 \equiv \frac{\sigma^{\widetilde{2}} - \sigma^{\widehat{2}}}{\sqrt{2}}, \quad g^3 \equiv \frac{\sigma^{\widetilde{1}} + \sigma^{\widehat{1}}}{\sqrt{2}}, \\
g^4 &\equiv \frac{\sigma^{\widetilde{2}} + \sigma^{\widehat{2}}}{\sqrt{2}}, \quad g^5 \equiv \sigma^{\widetilde{3}} + \sigma^{\widehat{3}},
\end{aligned}
\tag{3.2}
$$

and

$$
\begin{aligned}
\sigma^{\widetilde{1}} &\equiv \sin\theta_1 d\phi_1, \quad \sigma^{\widehat{1}} \equiv \cos 2\beta \sin\theta_2 d\phi_2 - \sin 2\beta d\theta_2, \quad \sigma^{\widetilde{2}} \equiv d\theta_1, \\
\sigma^{\widehat{2}} &\equiv \sin 2\beta \sin\theta_2 d\phi_2 + \cos 2\beta d\theta_2, \quad \sigma^{\widetilde{3}} \equiv \cos\theta_1 d\phi_1, \quad \sigma^{\widehat{3}} \equiv 2d\beta + \cos\theta_2 d\phi_2.
\end{aligned}
\tag{3.3}
$$

All other fields in the solution vanish. Near the horizon limit ($r \to 0$), this solution yields $AdS_5 \times T^{1,1}$ background geometry for type IIB superstring.

When the $D3$ branes are switched on, the resultant background is the K-S solution to type IIB supergravity [11]. This solution is composed of not only the ten-dimensional space-time metric, the self-dual five-form field strength $F_{(5)}$, but also the NS-NS- and R-R two-forms, $B_{(2)}$ and $C_{(2)}$, living only on $T^{1,1} \sim S^2 \times S^3$ [11],

$$
\begin{aligned}
ds^2_{10} &= h^{-1/2}(\tau)dx^2_{1,3} + h^{1/2}(\tau)ds^2_6, \\
\overline{F}_{(5)} &= \mathcal{F}_{(5)} + \widetilde{F}_{(5)}, \quad \mathcal{F}_{(5)} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dh^{-1}, \\
B_{(2)} &= \frac{g_s M\alpha'}{2}\left[f(\tau)g^1 \wedge g^2 + k(\tau)g^3 \wedge g^4\right], \\
F_{(3)} &= \frac{M\alpha'}{2}\left\{g^3 \wedge g^4 \wedge g^5 + d\left[F(\tau)\left(g^1 \wedge g^3 + g^2 \wedge g^4\right)\right]\right\}
\end{aligned}
\tag{3.4}
$$

In above equation $h(\tau)$ is the warp factor, ds^2_6 is the line element on the deformed conifold [11, 21],

$$
ds^2_6 = \frac{1}{2}(12)^{1/3}K(\tau)\left\{\frac{1}{3[K(\tau)]^3}\left[d\tau^2 + (g^5)^2\right]\right.
$$

$$
\begin{aligned}
&\quad + \sinh^2\left(\frac{\tau}{2}\right)\left[(g^1)^2+(g^2)^2\right] + \cosh^2\left(\frac{\tau}{2}\right)\left[(g^3)^2+(g^4)^2\right]\Big\}\,,\\
K(\tau) &= \frac{(\sinh 2\tau - 2\tau)^{1/3}}{2^{1/3}\sinh\tau}, \quad F(\tau) = \frac{\sinh\tau - \tau}{2\sinh\tau},\\
f(\tau) &= \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau - 1), \quad k(\tau) = \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau + 1),\\
h(\tau) &= C(g_s M)^2\frac{2^{2/3}}{4}\int_\tau^\infty dx \frac{x\coth x - 1}{\sinh^2 x}\left[\sinh(2x) - 2x\right]^{1/3}, \qquad (3.5)
\end{aligned}
$$

C being a normalization constant. This solution is non-singular since the apex of the conifold is resolved by the fractional brane fluxes [11].

At large τ, the radial coordinate τ is related to the radial coordinate r as $r^3 \sim 12^{1/2}e^{\tau}$. Consequently, the solution reduces to the Klebanov-Tseytlin (K-T) solution [22],

$$
\begin{aligned}
ds_{10}^2 &= h^{-1/2}(r)dx_{1,3}^2 + h^{1/2}(r)\left(dr^2 + r^2 ds^2_{T^{1,1}}\right),\\
\mathcal{F}_{(5)} &= N_{\text{eff}}(r)\omega_2\wedge\omega_3, \quad F_{(3)} = dC_{(2)} = c(r)\omega_3, \quad B_{(2)} = b(r)\omega_2,\\
h(r) &= \frac{27\pi(\alpha')^2}{4r^4}\left[g_s N + \frac{3}{2\pi}(g_s M)^2\ln\left(\frac{r}{r_0}\right) + \frac{3}{8\pi}(g_s M)^2\right],\\
\omega_2 &= \frac{1}{2}\left(g^1\wedge g^2 + g^3\wedge g^4\right) = \frac{1}{2}\left(\sin\theta_1 d\theta_1\wedge d\phi_1 - \sin\theta_2 d\theta_2\wedge d\phi_2\right),\\
\omega_3 &= g^5\wedge\omega_2, \quad \int_{S^2}\omega_2 = 4\pi, \quad \int_{S^3}\omega_3 = 8\pi^2,\\
b(r) &= \frac{3g_s M\alpha'}{2}\ln\left(\frac{r}{r_0}\right), \quad c(r) = \frac{M\alpha'}{2},\\
N_{\text{eff}}(r) &= N + \frac{3}{2\pi}g_s M^2\ln\left(\frac{r}{r_0}\right), \quad C_{(2)} = M\alpha'\beta\omega_2 \quad \text{(locally)}. \qquad (3.6)
\end{aligned}
$$

All other fields vanish. Obviously, this solution describes a deformed $AdS_5\times T^{1,1}$ geometry. Eq. (3.6) shows explicitly that there are M units of R-R three-form fluxes passing through the 3-cycle S^3 of $T^{1,1}$.

3.2 Fractional Brane as the Origin for Superconformal Anomaly

We use the brane probe technique [23] to show that presence of fractional branes in the brane configuration is the origin of the superconformal anomaly origin. The low-energy dynamics of a stack of Dp-brane system describes the dynamical behavior of open string modes trapped on the world-volume of Dp-branes and their interaction with bulk supergravity. For a single D-brane, the low-energy effective action consists of the Dirac-Born-Infeld action (in Einstein framework) [23]

$$
S_{\text{DBI}} = -\tau_p\int_{V_{p+1}} d^{p+1}x e^{(p-3)\phi/4}\sqrt{-\det\left[G_{\mu\nu} + e^{-\phi/2}\left(B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)\right]}. \qquad (3.7)
$$

and the Wess-Zumino (WZ) term,

$$
S_{\text{WZ}} = \mu_p\int_{V_{p+1}}\left(C\wedge e^{B+2\pi\alpha' F}\right)_{p+1}. \qquad (3.8)
$$

The former describes the interaction between a Dp-brane and NS-NS fields of type II superstring and the later is about the Dp-brane interacting with R-R fields. The parameters τ_p and μ_p are the tension and R-R charge of a Dp-brane, which are actually equal because of the BPS-saturation feature of Dp-brane,

$$\tau_p = \mu_p = \frac{1}{(2\pi)^p \alpha'^{(1+p)/2}}, \tag{3.9}$$

As for other fields, $F_{\mu\nu}$ is the strength of $U(1)$ gauge field living on the world-volume of Dp-branes; $G_{\mu\nu}$, $B_{\mu\nu}$ and $C_{\mu_1\mu_2\cdots\mu_n}$ are the pull-backs of ten-dimensional bulk metric G_{MN}, the antisymmetric NS-NS field $B_{(2)MN}$ and R-R fields $C_{M_1\cdots M_n}$ to the $p+1$-dimensional world-volume,

$$\begin{aligned} &G_{\mu\nu} = G_{MN}\partial_\mu X^M \partial_\nu X^N, \quad B_{\mu\nu} = B_{MN}\partial_\mu X^M \partial_\nu X^N, \\ &C_{\mu_1\mu_2\cdots\mu_n} = C_{M_1M_2\cdots M_n}\partial_{\mu_1} X^{M_1}\partial_{\mu_2} X^{M_2}\cdots\partial_{\mu_n} X^{M_n}, \quad n \leq p+1. \end{aligned} \tag{3.10}$$

With the DBI action and WZ term, we now demonstrate how the superconformal anomaly originates from the $D3$ fractional branes. First, when the transverse space like conifold has singularity, the closed string states consist of both the untwisted and twisted sectors [24]. The fractional Dp-branes frozen at the singular point of transverse space are identical to the $D(p+2)$-branes wrapped on two-cycle $\mathcal{C}_2$ like S^2 and the singular point can be considered as a vanishing two-cycle. The fields corresponding to the twisted string states emitted by fractional Dp-branes should locate at the singular point of target spacetime. Thus the twisted fields can always be decomposed into two parts, one part living on the $p+1$-dimensional world-volume of fractional Dp-brane, and the other on the blow-up of the vanishing two-cycle $\mathcal{C}_2$ [24],

$$V_{p+3} = V_{p+1}(x) \times \mathcal{C}_2, \quad B_{(2)} = B(x)\omega_2, \quad C_{(p+3)} = C_{(p+1)}(x) \wedge \omega_2. \tag{3.11}$$

In above equation the twisted scalar field $B(x)$ and the pull-back $C_{\mu_1\cdots\mu_{p+1}}$ of R-R antisymmetric tensor fields live on the world-volume of Dp-branes.

Second, we employ the D-brane probe technique to observe the dynamical behavior of the supersymmetric gauge theory implied from the D-brane dynamics. The basic idea of brane probe technique is the following [23]. In order to detect the dynamics of a supersymmetric $SU(N)$ gauge theory living on the world-volume of a stack of N coincident Dp-branes, one just places a probe brane of the same type near those coincident branes. This brane configuration will yield an $SU(N+1)$ supersymmetric gauge theory with spontaneous breaking of gauge symmetry $SU(N+1) \to SU(N) \times U(1)$. The justification for this is that the coordinates of transverse space are identified as scalar fields on Dp-brane world-volume. So when we move one brane some distance from the coincident Dp-branes, this means that the scalar fields get vacuum expectation values. If we consider the Coulomb branch of the supersymmetric $SU(N+1)$ gauge theory, the probe brane decouples from those coincident Dp-branes after symmetry breaking. However, according to the decoupling theorem, at the energy scale characterized by the distance of the probe brane away from those coincident branes, the coupling of supersymmetric $U(1)$ gauge theory on the

world-volume of probe brane should equal to the gauge coupling of $SU(N)$ gauge theory on the world-volume of those N coincident D-branes. Therefore, the probe brane action can provide us the information about the running of gauge coupling and shift of the θ-angle of the supersymmetric $SU(N)$ gauge theory living on the world-volume of coincident Dp-branes.

On the other hand, since a stack of N coincident Dp-branes modify the background space-time, the probe technique can be considered as a probe brane moving slowly in the supergravity background produced by the coincident branes to be probed. Therefore, we can use the DBI action and WZ term for the probe brane in the p-brane solution background produced by the N coincident Dp-branes to observe the dynamical behavior of a $p+1$-dimensional $SU(N)$ supersymmetric gauge theory. In this way one can easily get the quantum information of a supersymmetric gauge theory defined on the world-volume of those coincident Dp-branes.

Based on the brane probe technique, we substitute the fields in (3.11) into the DBI action (3.7) and the WZ term (3.8) for a single brane and expand them to the quadratic terms of the gauge field strength,

$$\begin{aligned}
& S_{\rm DBI}+S_{\rm WZ} \\
&= -\tau_{p+2}\int_{V_{p+1}\times\mathcal{C}_2} d^{p+3}x e^{(p-1)\phi/4}\sqrt{-\det\left[G_{\mu\nu}+e^{-\phi/2}\left(B_{ab}+2\pi\alpha' F_{\mu\nu}\right)\right]} \\
&\quad +\mu_{p+2}\int_{V_{p+1}\times\mathcal{C}_2}\left[C\wedge\left(e^{B+2\pi\alpha' F}\right)\right]_{(p+1)+2} \\
&= -\tau_{p+2}\int_{V_{p+1}} d^{p+1}x\, e^{(p-3)\phi/4}\sqrt{-\det\left(G_{\mu\nu}+2\pi\alpha' e^{-\phi/2}F_{\mu\nu}\right)}\int_{\mathcal{C}_2}B_{(2)} \\
&\quad +\mu_{p+2}\left[\int_{V_{p+1}}C_{(p+1)}\int_{\mathcal{C}_2}C_{(2)}+\frac{1}{2}(2\pi\alpha')^2\int_{V_{p+1}}C_{(p-3)}\wedge(F\wedge F)\int_{\mathcal{C}_2}C_{(2)}\right. \\
&\quad \left.+\int_{V_{p+1}}C_{(p+1)}\int_{\mathcal{C}_2}B_{(2)}+\frac{1}{2}(2\pi\alpha')^2\int_{V_{p+1}}C_{(p-3)}\wedge(F\wedge F)\int_{\mathcal{C}_2}B_{(2)}+\cdots\right] \\
&= S_{\rm brane-bulk}+S_{\rm gauge}; \\
& S_{\rm gauge} = -\alpha'\tau_p\int_{V_{p+1}} d^{p+1}x\, e^{(p-3)\phi/4}\sqrt{-\det\left(G_{\mu\nu}\right)}\left[-\frac{1}{4}\left(F^{\lambda\rho}F_{\lambda\rho}\right)\right]e^{-\phi}\int_{\mathcal{C}_2}B_{(2)} \\
&\quad +\frac{1}{2}\alpha'\mu_p\left[\int_{V_{p+1}}(F\wedge F)\wedge C_{(p-3)}\left(\int_{\mathcal{C}_2}C_{(2)}+\int_{\mathcal{C}_2}B_{(2)}\right)\right]+\cdots, \\
& \mu,\nu=0,\cdots,p;\quad \alpha,\beta=0,\cdots,p,a,b;
\end{aligned} \tag{3.12}$$

where $a,b=1,2$ are the indices on the vanishing 2-cycle $\mathcal{C}_2$. Note that in Eq. (3.12) we choose X^I=Constant ($I=p+1,\cdots,9$), and hence $G_{IJ}=0$. Recalling the relation between the gauge couplings and string couplings,

$$\frac{4i\pi}{g^2_{\rm YM}}+\frac{\theta^{(\rm CP)}}{2\pi}=ie^{-\phi}+\frac{C_{(0)}}{2\pi}, \tag{3.13}$$

and choosing $p=3$, we can see from Eq. (3.12) that it is the NS-NS- and R-R two-form fluxes $\int_{\mathcal{C}_2}B_{(2)}$, $\int_{\mathcal{C}_2}C_{(2)}$ carried by fractional branes through the shrunken 2-cycle that have

created the running of $U(1)$ gauge coupling and the shift of θ-angle. This exactly leads to the superconformal anomaly of the supersymmetric gauge theory on a stack of coincident D-branes. Therefore, we conclude that in a brane configuration the fractional branes frozen at the singular point of background space-time is the origin of superconformal anomaly of a supersymmetric gauge theory.

4. Manifestation of Superconformal Anomaly on K-S Solution

4.1 Breaking of $U(1)$ rotational and scale symmetries in transverse space by fractional $D3$-branes

In the following we focus on the brane configuration with N bulk $D3$-branes and M fractional $D3$-branes in target space-time $M^4 \times \mathcal{C}_6$ and analyze how the fractional brane affects the space-time symmetry of $AdS_5 \times T^{1,1}$.

The classical scale symmetry and chiral $U_R(1)$-symmetry of the $SU(N+M) \times SU(M)$ supersymmetric gauge theory living on the world-volume of $D3$-branes comes from the geometrical scale- and rotation transformation invariance of the transverse space,

$$x^I \longrightarrow \mu e^{i\alpha} x^I, \quad 4 \leq I \leq 10. \tag{4.1}$$

Now let us observe how the above scale- and $SO(2) \cong U(1)$ rotation symmetries are reflected in the K-S solution. First, in the case with no fractional branes, the near-horizon ($r \to 0$) limit of the three-brane solution (3.1) is $AdS_5 \times T^{1,1}$,

$$\begin{aligned}
ds^2_{10} &= \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2 + L^2 ds^2_{T^{1,1}} \\
ds^2_{T^{1,1}} &= \frac{1}{9}\left(2d\beta + \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right) \\
F_{(5)} &= \mathcal{F}_{(5)} + {}^\star\mathcal{F}_{(5)} = \frac{1}{2}\pi\alpha'^2 N\left[\omega_2 \wedge \omega_3 + {}^\star(\omega_2 \wedge \omega_3)\right] \\
&= \frac{r^3}{g_s L^4}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dr + \frac{L^4}{27 g_s}g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5, \\
L^2 &= \frac{3\sqrt{3\pi g_s N}\alpha'}{2}.
\end{aligned} \tag{4.2}$$

This solution shows explicitly that the scale symmetry is trivially present in the transverse space part and the $U(1)$ symmetry is reflected in the invariance under the β-angle rotation, $\beta \longrightarrow \beta + \alpha$. Then we switch on fractional branes, the solution (3.6) shows that the metric ceases to be $AdS_5 \times T^{1,1}$. The reason for this is that $h(r)$ and $F_{(5)}$ get logarithmic dependence on the radial coordinate,

$$\begin{aligned}
h(r) &\sim \frac{1}{r^4}\left[C_1 + C_2 \ln\frac{r}{r_0}\right], \\
\overline{F}_5 &\sim \left[N + C_3 \ln\frac{r}{r_0}\right]\left[\omega_2 \wedge \omega_3 + {}^\star(\omega_2 \wedge \omega_3)\right],
\end{aligned} \tag{4.3}$$

where the coefficients C_i $(i = 1, 2, 3)$ can be extracted out from the solution (3.6). Further, the background fields $B_{(2)MN}$ and $C_{(2)MN}$ are generated by the fractional brane fluxes.

In Ref. [10], it was analyzed that the non-invariance of $C_{(2)}$ under the β-angle rotation leads to the chiral R-symmetry anomaly in $\mathcal{N} = 1$ supersymmetric $SU(N + M) \times SU(N)$ gauge theory. We make a similar analysis to show that the non-invariance of $B_{(2)}$ under scale transformation in transverse space results in the scale anomaly. The $B_{(2)}$ in Eq. (3.4) under the scale transformation $r \to \mu r$ transforms as

$$B_{(2)} \longrightarrow B_{(2)} + \frac{3g_s M\alpha'}{2}\omega_2 \ln \mu. \tag{4.4}$$

According to the relation between string coupling and gauge couplings of the $\mathcal{N} = 1$ supersymmetric $SU(N + M) \times SU(N)$ gauge theory [11, 12],

$$\frac{1}{g_1^2} + \frac{1}{g_2^2} \sim e^{-\phi} \sim \frac{1}{g_s}, \quad \frac{1}{g_1^2} - \frac{1}{g_2^2} \sim e^{-\phi}\left[\int_{S^2} B_{(2)} - \frac{1}{2}\right], \tag{4.5}$$

one has

$$\frac{1}{g_1^2} \sim \frac{1}{2g_s}\left[\int_{S^2} B_{(2)} - \frac{1}{2}\right], \quad \frac{1}{g_1^2} \sim \frac{1}{2g_s}\left[-\int_{S^2} B_{(2)} + \frac{3}{2}\right]. \tag{4.6}$$

A comparison between (4.4) and (4.6) shows that the variation of $B_{(2)}$ under scale transformation is relevant to β-functions for the $SU(N + M) \times SU(N)$ gauge couplings at the IR fixed points of the $\mathcal{N} = 1$ $SU(N) \times SU(N)$ supersymmetric gauge theory [11],

$$\frac{d}{d(\ln \mu)} \frac{8\pi^2}{g_1^2(\mu)} \sim 3M, \quad \frac{d}{d(\ln \mu)} \frac{8\pi^2}{g_2^2(\mu)} \sim -3M. \tag{4.7}$$

This exactly lead to the scale anomaly coefficients shown in Eq. (2.21).

4.2 Supersymmetry Breaking due to Modification on $AdS_5 \times T^{1,1}$ by Fractional Branes

In this subsection we use the result of Ref. [21] to emphasize how one-half of supersymmetries break in type IIB supergravity due to the modification on space-time background by fractional branes. The standard method of investigating supersymmetry breaking produced by fractional branes is to check the Killing spinor equations obtained from supersymmetry transformations for the fermionic fields of type IIB supergravity in the K-S solution background (3.4) and count the number of Killing spinors. The bosonic field content of type IIB supergravity consists of the dilaton field ϕ, the metric G_{MN} and the second-rank antisymmetric tensor field $B_{(2)MN}$ in the NS-NS sector, the axion field $C_{(0)}$, the two-form potential $C_{(2)MN}$ and four-form potential $C_{(4)MNPQ}$ with self-dual field strength in the R-R sector. The fermionic fields are left-handed complex Weyl gravitino Ψ_M and right-handed complex Weyl dilatino Λ, $\Gamma^{11}\Psi_M = -\Psi_M$, $\Gamma^{11}\Lambda = \Lambda$. The supersymmetry transformations for the fermionic fields read [25],

$$\delta\Lambda = \frac{i}{\kappa_{10}}\Gamma^M \epsilon^\star P_M - \frac{i}{24}\Gamma^{MNP}\epsilon G_{(3)MNP} + \text{fermions relevant terms},$$

$$
\begin{aligned}
\delta\Psi_M =& \frac{1}{\kappa_{10}}\left(D_M - \frac{1}{2}iQ_M\right)\epsilon + \frac{i}{480}\Gamma^{PQRST}\Gamma_M\epsilon\widehat{F}_{(5)PQRST} \\
&+\frac{1}{96}\left(\Gamma_M{}^{NPQ}G_{(3)NPQ} - 9\Gamma^{NP}G_{(3)MNP}\right)\epsilon^\star + \text{fermions relevant terms}, \\
&M, N, \cdots = 0, 1, \cdots, 9.
\end{aligned} \tag{4.8}
$$

In above equation, the supersymmetry transformation parameter ϵ is a left-handed complex Weyl spinor, $\Gamma^{11}\epsilon = -\epsilon$, and other quantities are listed as the following [25, 26],

$$
\begin{aligned}
P_M &= f^2\partial_M B, \quad Q_M = f^2\mathrm{Im}\left(B\partial_M B^\star\right), \quad B \equiv \frac{1+i\tau}{1-i\tau}, \\
f^{-2} &= 1 - BB^\star, \quad \tau = C_{(0)} + ie^{-\phi}, \\
G_{(3)MNP} &= f\left(\widehat{F}_{(3)MNP} - B\widehat{F}^\star_{(3)MNP}\right), \\
\widehat{F}_{(3)MNP} &= 3\partial_{[M}A_{(2)NP]}, \quad A_{(2)MN} \equiv C_{(2)MN} + iB_{(2)MN}, \\
\widehat{F}_{(5)MNPQR} &= F_{(5)MNPQR} - \frac{1}{8}\times 10\,\kappa_{10}\mathrm{Im}\left(A_{[(2)MN}\widehat{F}_{(3)PQR]}\right) \\
&= F_{(5)MNPQR} - \frac{5}{4}\kappa_{10}C_{[(2)MN}H_{(3)PQR]} + \frac{5}{4}\kappa_{10}B_{[(2)MN}F_{(3)PQR]}, \\
F_{(5)MNPQR} &= 5\partial_{[M}C_{(4)NPQR]}, \\
D_M\epsilon &= \partial_M\epsilon + \frac{1}{4}\omega_M{}^{AB}\Gamma_{AB}\epsilon.
\end{aligned} \tag{4.9}
$$

To highlight the effects of fractional branes, we first consider the supersymmetry transformation in the background without fractional branes, i.e., the background (3.1) furnished by three-brane solution whose near-horizon limit is $AdS_5 \times T^{1,1}$. The supersymmetry transformations for the fermionic fields in the $AdS_5 \times T^{1,1}$ background read

$$
\begin{aligned}
\delta\Lambda &= 0, \\
\delta\Psi_M &= \frac{1}{\kappa_{10}}\left(\partial_M + \frac{1}{4}\omega_M{}^{AB}\Gamma^{AB}\right)\epsilon + \frac{i}{480}\Gamma^{PQRST}F_{(5)PQRST}\Gamma_M\epsilon = 0.
\end{aligned} \tag{4.10}
$$

The first equation $\delta\Lambda = 0$ is trivially satisfied in the background (4.2). The Killing spinor equation $\delta\Psi_M = 0$ yields [21]

$$
\begin{aligned}
\epsilon &= r^{\Gamma_\star/2}\left[1 + \frac{\Gamma_r}{2R^2}x^\mu\Gamma_\mu\left(1 - \Gamma_\star\right)\right]\epsilon_0, \\
\Gamma_\star &\equiv i\Gamma_{x_0}\Gamma_{x_1}\Gamma_{x_2}\Gamma_{x_3}, \quad \Gamma_\star^2 = 1,
\end{aligned} \tag{4.11}
$$

where ϵ_0 is an arbitrary constant spinor in ten dimensions but constrained by

$$
\Gamma_{g_1g_2}\epsilon_0 = \epsilon_0, \quad \Gamma_{g_3g_4}\epsilon_0 = -\epsilon_0. \tag{4.12}
$$

In Eqs. (4.11) and (4.12) the coordinates of $AdS_5 \times T^{1,1}$ are used to denote the components of Γ-matrices for clarity. The constraint (4.12) on ϵ_0 leads to $\Gamma_{g_1g_2}\epsilon = \epsilon$, $\Gamma_{g_3g_4}\epsilon = -\epsilon$. Therefore, the Killing spinor ϵ has eight independent components.

The Killing spinor (4.11) is actually a unified expression for the following two types of Killing spinors [21, 27],

$$\epsilon_+ = r^{1/2}\epsilon_{0+}, \quad \epsilon_- = r^{-1/2}\epsilon_{0-} + \frac{r^{1/2}}{L^2}\Gamma_r x^\mu \Gamma_\mu \epsilon_{0-},$$
$$\epsilon_{0\pm} = \frac{1}{2}(1 \pm \Gamma_\star)\,\epsilon_0, \quad \Gamma_\star \epsilon_{0\pm} = \pm\epsilon_{0\pm}. \tag{4.13}$$

Eq. (4.13) shows that ϵ_+ is independent of the coordinates on $D3$-brane world-volume and is a right-handed eigenspinor of $\Gamma_\star$, i.e., $\Gamma_\star\epsilon_+ = \epsilon_+$. It thus represents $\mathcal{N} = 1$ Poincáre supersymmetry in the dual supersymmetric gauge theory. Further, ϵ_- depends on x^μ linearly and is not an eigenspinor of $\Gamma_\star$. So it characterizes $\mathcal{N} = 1$ conformal supersymmetry of the dual supersymmetric gauge theory in four dimensions.

When the M fractional branes are switched on, the space-time background is described by the K-S solution (3.4). The Killing spinor equations for dilatino and gravitino become

$$\begin{aligned}
\delta\Lambda &= -\frac{i}{24}\widehat{F}_{(3)MNP}\Gamma^{MNP}\epsilon = 0,\\
\delta\Psi_M &= \frac{1}{\kappa}\left(\partial_M + \frac{1}{4}\omega_M{}^{AB}\Gamma^{AB}\right)\eta + \frac{i}{480}\Gamma^{PQRST}\Gamma_M\epsilon\widetilde{F}_{(5)PQRST}\\
&\quad + \frac{1}{96}\left(\Gamma_M{}^{NPQ}\widehat{F}_{(3)NPQ} - 9\Gamma^{NP}\widehat{F}_{(3)MNP}\right)\epsilon^\star = 0.
\end{aligned} \tag{4.14}$$

The same procedure with the metric (3.4) as in the case with no fractional brane leads to the following Killing spinor [21]

$$\epsilon = h^{-1/8}(\tau)\exp\left(-\frac{\alpha}{2}\Gamma_{g_1g_2}\right)\epsilon_0, \tag{4.15}$$

where α is determined by $\sin\alpha = -1/\cosh\tau$, $cos\alpha = \sinh\tau/\cosh\tau$. ϵ_0 is a constant spinor constrained by [21],

$$\Gamma_\star\epsilon_0 = -i\epsilon_0, \quad \Gamma_{g_1g_2}\epsilon_0 = -\Gamma_{g_3g_4}\epsilon_0, \quad \Gamma_{rg_5}\epsilon_0 = -i\epsilon_0. \tag{4.16}$$

Hence the Killing spinor ϵ satisfies

$$\Gamma_\star\epsilon = -i\epsilon, \quad \Gamma_{g_1g_2}\epsilon = -\Gamma_{g_3g_4}\epsilon = i\left(\cos\alpha + \sin\alpha\Gamma_{g_1g_3}\right)\epsilon, \quad \Gamma_{rg_5}\epsilon = -i\epsilon. \tag{4.17}$$

These three constraints determines that ϵ has only four independent components. In particular, ϵ is independent of the coordinate on the world-volume of $D3$-branes. Therefore, the four-component Killing spinor ϵ means the only existence of $\mathcal{N} = 1$ Poinráe supersymmetry in the dual supersymmetric gauge theory. This indicates that the conformal supersymmetry in the dual four-dimensional supersymmetric gauge theory collapses due to the presence of fractional $D3$-branes.

5. Superconformal anomaly as spontaneously breaking of local supersymmetry in gauged AdS_5 supergravity and super-Higgs Mechanism

5.1 Generality

The discussions in last section have shown that in comparison with $AdS_5 \times T^{1,1}$ case, the K-S solution background fails to preserve some of local symmetries in type IIB supergravity.

We choose the K-S solution as a classical vacuum configuration for type IIB supergravity and observe the theory around such a background. In the case without fractional branes, the near-horizon limit of the three-brane solution is $AdS_5 \times T^{1,1}$ (cf. (4.2)). However, a gravitational system has geometrical meaning, expanding ten-dimensional type IIB supergravity around $AdS_5 \times T^{1,1}$ is actually a process of performing spontaneous compactification of type IIB supergravity [28, 29, 30] on $T^{1,1}$. The resultant theory after compactification should be $\mathcal{N} = 2$ five-dimensional $U(1)$ gauged AdS_5 supergravity coupled with $\mathcal{N} = 2$ $SU(2) \times SU(2)$ Yang-Mills vector multiplets and several Betti tensor supermultiplets, whose origin is due to the nontrivial topology of $T^{1,1}$ [31]. The local symmetries in $\mathcal{N} = 2$ $U(1)$ gauged AdS_5 supergravity consist of $\mathcal{N} = 2$ supersymmetry, $SO(2,4)$ isometry symmetry and $U(1)$ gauge symmetry.

When fractional branes switch on, the space-time background becomes the K-S solution and its UV limit, the K-T solution (3.6) is a deformed $AdS_5 \times T^{1,1}$. Consequently, the isometry symmetry of the deformed $AdS_5 \times T^{1,1}$ and the supersymmetry it preserves is less than that exploited from $AdS_5 \times T^{1,1}$. Therefore, the compactified theory obtained from the compactification of type IIB supergravity on the deformed $T^{1,1}$, i.e., a certain five-dimensional gauged supergravity, should possess less local symmetries than those extracted from $AdS_5 \times T^{1,1}$. This means that some of local symmetries break spontaneously in the gauged AdS_5 supergravity since the symmetry loss originates from the vacuum configuration. Further, as is well known, a physical consequence of spontaneous breaking of local symmetry is the occurrence of the Higgs mechanism. To show this phenomenon clearly, just like what usually done in gauge theory, we reparametrize the field variable and "shift" the vacuum configuration described by the K-T solution back to $AdS_5 \times T^{1,1}$. The essence of this operation is performing local symmetry transformation and transferring the non-symmetric feature of vacuum configuration to the classical action. Then when we expand type IIB supergravity around $AdS_5 \times T^{1,1}$ with newly defined field variables, the action of the five-dimensional gauged supergravity should lose some of local symmetries and the graviton multiplet in gauged AdS_5 supergravity should acquire a mass by eating a Goldstone multiplet relevant to NS-NS- and R-R two-form fields in the K-S solution. In this way, we reveal how the super-Higgs mechanism due to the spontaneous breaking of local supersymmetry in the gauged AdS_5 supergravity occurs. This phenomenon was actually somehow noticed in the Kaluza-Klein supergravity [32]: when the internal manifold is deformed or squashed, it keeps less symmetries for the compactified theory than the undeformed internal manifold. This is the so-called "space invader" scenario and can be naturally given an interpretation in terms of spontaneous breaking of local symmetry [32].

Based on above analysis, we first consider the case without fractional brane and observe type IIB supergravity in $AdS_5 \times T^{1,1}$ background. This will lead to $\mathcal{N} = 2$ $U(1)$ gauged AdS_5 supergravity coupled to $SU(2) \times SU(2)$ Yang-Mills vector multiplets and Betti scalar-, vector- and tensor multiplets [31] in which the graviton supermultiple is massless. Then we go to the case with fractional branes and see how the graviton multiplet becomes massive due to the spontaneous breaking of local supersymmetry in gauged AdS_5 supergravity.

5.2 Type IIB supergravity in $AdS_5 \times T^{1,1}$ Background and Gauged AdS_5 Supergravity

We first give a brief review on the compactification of type IIB supergravity on $T^{1,1}$ [31]. In general, when performing compactification of a certain D-dimensional supergravity on $AdS_{D-d} \times K^d$, one should first linearize the classical equation of motion of the field function $\Phi(x,y) \equiv \Phi^{\{J\}[\lambda]}(x,y)$ in $AdS_{D-d} \times K^d$ background. In above, $K^d = G/H$ is certain d-dimensional compact Einstein manifold, x and y are the coordinates on AdS_{D-d} and K^d, respectively; $\{J\}$ and $[\lambda]$ denote the representations of local Lorenz groups $SO(2, D-d-1)$ and $SO(d)$ realized on the field function. The next step is to expand y-dependent part of the field function in terms of H-harmonics on K^d, which are representations of the group G branched with respect to its subgroup H. If the internal manifold is $K^d = S^d = SO(d+1)/SO(d)$, the maximally symmetric Einstein manifold, the expansion procedure works straightforwardly since H is $SO(d)$, which is precisely the local Lorenz group of the d-dimensional internal manifold. However, if the internal manifold is a less symmetric one, and H is a subgroup of $SO(d)$. Then the field function representations of $SO(d)$ are usually reducible with respect to H. Therefore, only those H-harmonics that are identical to the $SO(d)$ field function representations branched by H can contribute to the expansion of field functions on K^d.

The compactification of type IIB supergravity on $T^{1,1}$ is exactly this case. $T^{1,1}$ is the coset space $G/H = [SU(2) \times SU(2)]/U_{\rm H}(1)$ and the generator of $U_{\rm H}(1)$ is the sum $T^3 + \widehat{T}^3$ of two diagonal generators T^3 and $\widehat{T}^3$ of $SU(2) \times SU(2)$. The harmonics on $T^{1,1}$ are representations of $SU(2) \times SU(2)$ labeled by the weight $\{\nu\} = (j, l)$,

$$Y(y) \equiv \left(\left[Y^{(j,l,r)}(y)\right]^m\right). \tag{5.1}$$

where $m = 1, \cdots, (2j+1) \times (2l+1)$, which labels the representation of $SU(2) \times SU(2)$, and r is the quantum number for the $U(1)$ group whose generator is $T_3 - \widehat{T}_3$. The representations (j, l) are reducible with respect to the subgroup $U_{\rm H}(1)$ and hence decompose into a direct sum of fragments graded by the $U_{\rm H}(1)$-charge q_i,

$$\left[Y^{(j,l,r)}(y)\right]^m = \bigoplus_i \left[Y^{(j,l,r)}(y)\right]^m_{q_i} = \begin{pmatrix} \left[Y^{(j,l,r)}(y)\right]^m_{q_1} \\ \left[Y^{(j,l,r)}(y)\right]^m_{q_2} \\ \vdots \\ \left[Y^{(j,l,r)}(y)\right]^m_{q_N} \end{pmatrix}. \tag{5.2}$$

The irreducible representations $\left[Y^{(j,l,r)}(y)\right]^m_{q_i}$ are called $U_{\rm H}(1)$-harmonics on $T^{1,1}$.

On the other hand, the field function $\Phi^{\{J\}[\lambda]}(x,y)$ on $AdS_5 \times T^{1,1}$ belongs to a certain representation of the local Lorentz group $SO(2,4) \times SO(5)$, $\{J\}$, $[\lambda]$ denoting representation wights for $SO(2,4)$ and $SO(5)$, respectively. The $U_{\rm H}(1)$ is a subgroup of $SO(5)$ because the representation of its generator is naturally embedded into the representation of $SO(5)$ generators [33]. The $SO(5)$ representations $[\lambda]$ furnished by $\left[X^{[\lambda]}(y)\right]^n$, the y-dependent part of $\Phi^{\{J\}[\lambda]}(x,y)$, are also reducible with respect to $U_{\rm H}(1)$, where n labels

the representation of $SO(5)$ in field function space. The field function representation space thus decomposes into a direct sum of irreducible subspaces labeled by the $U_{\rm H}(1)$-charge q_ξ,

$$\left[X^{[\lambda]}(y)\right]^n = \bigoplus_{\xi=1}^{K}\left[X^{[\lambda]}(y)\right]^n_{q_\xi} = \begin{pmatrix} \left[X^{[\lambda]}(y)\right]^n_{q_1} \\ \left[X^{[\lambda]}(y)\right]^n_{q_2} \\ \vdots \\ \left[X^{[\lambda]}(y)\right]^n_{q_K} \end{pmatrix}. \tag{5.3}$$

The irreducible representations $\left[X^{[\lambda]}(y)\right]^n_{q_\xi}$ in above equation are called the $SO(5)$ harmonics. The field function admits a natural expansion in terms of $SO(5)$ harmonics. Therefore, an $U_{\rm H}$-harmonics Y can contribute to the expansion of a field function on $T^{1,1}$ only when it is identical to (or contains) certain $SO(5)$ harmonics X. A detailed analysis on how the $SU(2)\times SU(2)$ representations branched with respect to $U_{\rm H}(1)$ appear in the decomposition of the $SO(5)$ field function representations with respect to $U_{\rm H}(1)$ is shown in Ref. [31].

Once the expansion of field functions $\Phi^{\{J\}[\lambda]}(x,y)$ in terms of those admissible $U_{\rm H}(1)$-harmonics on $T^{1,1}$ is known. That is [31],

$$\Phi^{\{J\}[\lambda]}(x,y) = \left(\Phi^{\{J\}}_{ab\cdots}(x,y)\right)^{[\lambda]} = \bigoplus_{i=1}^{N}\left(\Phi^{\{J\}}_{q_i}\right)^{[\lambda]} = \begin{pmatrix} \Phi^{\{J\}}_{q_1}(x,y) \\ \Phi^{\{J\}}_{q_2}(x,y) \\ \vdots \\ \Phi^{\{J\}}_{q_N}(x,y) \end{pmatrix},$$
$$\Phi^{\{J\}}_{q_i}(x,y) = \sum_{j,l}\sum_{m}\sum_{r}\Phi^{\{J\}(j,l,r)}_{q_i m}(x)\left[Y^{(j,l,r)}(y)\right]^m_{q_i}, \tag{5.4}$$

one substitutes it into the linearized equations of motion of type IIB supergravity in $AdS_5\times T^{1,1}$ background,

$$\left(K_x^{\{J\}} + K_y^{[\lambda]}\right)\Phi^{\{J\}[\lambda]}(x,y) = 0. \tag{5.5}$$

In Eqs. (5.4) and (5.5), a, b are the indices for the $[\lambda]$-representation, $K_x^{\{J\}}$ and $K_y^{[\lambda]}$ are the kinetic operators of type IIB supergravity on AdS_5 and $T^{1,1}$, respectively. There are three types kinetic operators $K_y^{[\lambda]}$ in supergravity: the Hodge-de Rahm- and Laplacian operators acting on scalar-, vector- and antisymmetric fields, the Dirac and Rarita-Schwinger operator acting on the fermionic fields and the Lichnerowicz operator on the symmetric rank-two graviton field. All these three types of operators are (or are related to) certain Laplace-Beltrami operators on $T^{1,1}$. Thus the action of $K_y^{[\lambda]}$ on $U_{\rm H}(1)$-harmonics gives [31]

$$K_y^{[\lambda]}\left[Y^{(j,l,r)}(y)\right]^m_{q_i} = M^{(j,l,r)}_{ik}\left[Y^{(j,l,r)}(y)\right]^m_{q_k}. \tag{5.6}$$

This equation together with the linearized equation of motion (5.5) and $U_{\rm H}(1)$-harmonic expansion (5.4) of field functions determines that $M^{(j,l,r)}_{ij}$ are mass matrices for the K-K particle tower $\Phi^{\{J\}(j,l,r)}_{q_k m}(x)$ in AdS_5 space,

$$\left(\delta_{ik}K_x^{\{J\}} + M^{(j,l,r)}_{ik}\right)\Phi^{\{J\}(j,l,r)}_{q_k m}(x) = 0. \tag{5.7}$$

Therefore, the zero modes of the kinetic operator $K_y^{[\lambda]}$ constitute the field content of $\mathcal{N}=2$ $U(1)$ gauged AdS_5 supergravity coupled with $SU(2)\times SU(2)$ Yang-Mills vector supermultiplets as well as some Betti tensor multiplets [31].

We then turn to the explicit form of $AdS_5 \times T^{1,1}$. As a standard step in Kaluza-Klein theory, to perform the compactification on $T^{1,1}$, one should write the metric (3.1) in the Kaluza-Klein metric form,

$$\begin{aligned} ds^2 &= h^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2})(r)dr^2 \\ &+ h^{1/2}(r)r^2\left\{\frac{1}{9}\left(g^5-2A\right)^2+\frac{1}{6}\left[\sum_{r=1}^{2}(g^r-K^{ar}W^a)^2+\sum_{s=3}^{4}\left(g^s-L^{bs}\widetilde{W}^b\right)^2\right]\right\} \end{aligned} \tag{5.8}$$

In Eq. (5.8), K^{ar} and L^{bs} are the components of Killing vectors K^a and L^b on $T^{1,1}$, and they convert into the generators of $SU(2)\times SU(2)$ gauge group in the resultant $\mathcal{N}=2$ $U(1)$ gauged supergravity coupled with six Yang-Mills vector supermultiplets,

$$[K^{a_1}, K^{a_2}] = i\epsilon^{a_1a_2a_3}K^{a_3}, \quad [L^{b_1}, L^{b_2}] = i\epsilon^{b_1b_2b_3}L^{b_3}; \tag{5.9}$$

$W^a = W^a{}_\alpha dx^\alpha$ and $\widetilde{W}^b = \widetilde{W}^b{}_\alpha dx^\alpha$ are the corresponding six $SU(2)$ gauge fields; $A = A_\alpha dx^\alpha$ is the $U(1)$ gauge field corresponding to the isometric symmetry $U(1)$, and it constitutes an $\mathcal{N}=2$ supermultiplets with the graviton $h_{\alpha\beta}$ and gravitini ψ^i_α, $i=1,2$, in the gauged AdS_5 supergravity.

As a straightforward consequence of using the K-K metric (5.8), the self-dual five-form in Eq. (3.1) should be modified to keep its self-duality with respect to the K-K metric [10],

$$\begin{aligned} \overline{F}_5 &= d\overline{C}_4 = \frac{1}{g_s}\partial_r h^{-1}(r)dx^0\wedge dx^1\wedge dx^2\wedge dx^3\wedge dr \\ &+\frac{L^4}{27}\left[\chi\wedge g^1\wedge g^2\wedge g^3\wedge g^4 - dA\wedge g^5\wedge dg^5 + \frac{3}{L}\left(^{\star_5}dA\right)\wedge dg^5\right]. \end{aligned} \tag{5.10}$$

Locally, there exists

$$\begin{aligned} \overline{C}_4 &= \frac{1}{g_s}h^{-1}(r)dx^0\wedge dx^1\wedge dx^2\wedge dx^3 \\ &+ \frac{2L^4}{27}\left[\beta g^1\wedge g^2\wedge g^3\wedge g^4 - \frac{1}{2}A\wedge g^5\wedge dg^5 + \frac{3}{2r}h^{-1/4}(r)\left(^{\star_5}dA\right)\wedge dg^5\right], \end{aligned} \tag{5.11}$$

where $\chi = g^5 - 2A$, $\star_5$ is the five-dimensional Hodge dual operator defined with respect to AdS_5 metric

$$ds^2_{\mathrm{AdS_5}} = g^{(0)}_{\alpha\beta}dx^\alpha dx^\beta = \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2, \quad \alpha,\beta = 0,1,\cdots,4. \tag{5.12}$$

Obviously, both the K-K metric (5.8) and $\overline{F}_5$ are gauge invariant under local $U(1)$ gauge transformation in gauged AdS_5 supergravity,

$$\beta \to \beta + \varphi, \quad A \to A + d\varphi. \tag{5.13}$$

$\overline{F}_5$ satisfies the Bianchi identity $d\overline{F}_5 = 0$, which determines A is a massless vector fields in AdS_5 space, i.e., $d^{\star_5}dA = 0$.

Finally, we briefly mention how the graviton supermultiplet in the gauged AdS_5 supergravity comes from the compactification of type IIB supergravity on $T^{1,1}$ [31]. First, $h_{\alpha\beta}(x)$ is the zero-mode in the scalar $U_{\rm H}(1)$-harmonic expansion of $h_{\alpha\beta}(x,y)$, which is the AdS_5 space-time component of ten-dimensional graviton $h_{MN}(x,y)$; A_α is the linear combination of two zero-modes in the vector $U_{\rm H}(1)$-harmonic expansion of $h_{\alpha a}$ and $C_{(4)\alpha abc}$, which are the crossing components on AdS_5 and $T^{1,1}$ of the graviton $h_{MN}(x,y)$ and the R-R four form potential $C_{(4)MNPQ}(x,y)$, respectively; ψ_α comes from the massless mode in the spinor $U_{\rm H}(1)$-harmonics expansion of ψ_α, which is the AdS_5 component of ten-dimensional gravitino ψ_M [31]. The quadratic action for the graviton supermultiplet of $\mathcal{N} = 2$ $U(1)$ gauged AdS_5 supergravity can be straightforwardly extracted out from the compactification of type IIB supergravity on $AdS_5 \times T^{1,1}$,

$$S = \frac{1}{\kappa_5^2}\int d^5x\sqrt{-g}\left(-\frac{1}{2}R - \frac{1}{2}\overline{\psi}^i_\alpha\gamma^{\alpha\beta\gamma}D_\beta\psi_{i\gamma} - \frac{3L^2}{32}F^{\alpha\beta}F_{\alpha\beta} - \frac{6}{L^2} + \cdots\right). \tag{5.14}$$

It shows that the graviton multiplet $(A_\alpha, \psi^i_\alpha, h_{\alpha\beta})$ is massless.

Now we come to the stage of revealing the dual of superconformal anomaly in the $\mathcal{N} = 1$ $SU(N+M) \times SU(N)$ supersymmetric gauge theory with two flavors in the bifundmental representation of gauge group.

5.3 Dual of Chiral $U_R(1)$-symmetry anomaly $\partial_\mu j^\mu$

In the following we review briefly how the dual of chiral $U_R(1)$ anomaly of $\mathcal{N} = 1$ $SU(N+M) \times SU(N)$ gauge theory was found in Ref. [10] from type IIB supergravity in the K-S solution background.

When there is no fractional brane, the K-K metric and the five-form field shown in Eqs. (5.8) and (5.10) are $U(1)$ gauge invariant after the compactification on $T^{1,1}$ is performed. The graviton multiplet of gauged AdS_5 supergravity is massless. When fractional branes are present, both the K-K metric (5.8) and the five-form potential (5.10) get deformed, but they still keep invariant under $U(1)$ gauge transformation. However, Eq. (3.4) shows that the field strength $F_{(3)}$ of R-R two-form $C_{(2)}$ arises in the background solution. Since $F_{(3)}$ depends linearly on the angle β,

$$F_{(3)} = \frac{M\alpha'}{2}\omega_3 = \frac{M\alpha'}{2}g^5 \wedge \omega_2 = \frac{M\alpha'}{4}g^5 \wedge \left(g^1 \wedge g^2 + g^3 \wedge g^4\right), \tag{5.15}$$

it is not invariant under the rotation of β. As what usually done in dealing with the spontaneous breaking of gauge symmetry, we shift $F_{(3)}$ as

$$\overline{F}_{(3)} = \frac{M\alpha'}{2}\left(g^5 + 2\partial_\alpha\theta dx^\alpha\right) \wedge \omega_2. \tag{5.16}$$

by introducing a new field,

$$\theta \equiv F\int_{S^2} C_2, \tag{5.17}$$

where F is a field function in ten dimensions. The shifted $\overline{F}_{(3)}$ is obviously invariant under the gauge transformation

$$\beta \to \beta + \varphi, \quad \theta \to \theta - \varphi. \tag{5.18}$$

Further, $\overline{F}_{(3)}$ can be re-expressed in terms of a gauge invariant $\chi \equiv g^5 - 2A$ and a newly defined vector field W_α,

$$\begin{aligned}\overline{F}_3 &= \frac{M\alpha'}{2}\left(\chi + 2W_\alpha dx^\alpha\right) \wedge \omega_2 = \overline{F}_3^{(0)} + M\alpha' W_\alpha dx^\alpha \wedge \omega_2;\\ W_\alpha &\equiv A_\alpha + \partial_\alpha \theta, \quad \overline{F}_3^{(0)} \equiv \frac{M\alpha'}{2} \wedge \omega_2.\end{aligned} \tag{5.19}$$

The vector field W_α and the $U(1)$ gauge field A_α have the same field strength $F_{\alpha\beta}$, but W_α has got a longitudinal component. We then consider type IIB supergravity in the symmetric vacuum configuration furnished by the K-K metric (5.8), the self-dual five-form (5.10) and R-R three-form field strength $\overline{F}_3^{(0)}$ defined in (5.19). The Einstein-Hilbert- and the R-R three-form terms in the classical action of type IIB supergravity, gives [10]

$$\begin{aligned}S_{\mathrm{IIB}} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\left[ER - \frac{1}{2}F_3 \wedge {}^\star F_3 + \cdots\right]\\ &= \frac{1}{2\kappa_{10}^2}\int d^{10}xE\left[-\frac{1}{9}h^{1/2}(r)r^2 F^{\alpha\beta}F_{\alpha\beta} - \left(\frac{3M\alpha'}{h^{1/2}(r)r^2}\right)^2 W^\alpha W_\alpha + \cdots\right].\end{aligned} \tag{5.20}$$

This ten-dimensional action implies that the $U(1)$ gauge field in gauged AdS_5 supergravity, which comes from the compactification of type IIB supergravity on $T^{1,1}$, obtain a mass proportional to the fluxes carried by fractional branes. This phenomenon is exactly the Higgs effect in the following sense: when fractional branes are present, the classical vacuum configuration is $U(1)$ symmetric. After the classical solution is modified to make it gauge invariant, the classical action of gauged AdS_5 supergravity around the symmetric vacuum configuration should lose gauge symmetry and the $U(1)$ gauge field acquires a mass.

5.4 Dual of Scale Anomaly $\theta^\mu{}_\mu$

In this subsection we investigate the gravity dual of scale anomaly in a similar way as looking for the dual of chiral anomaly. It should also correspond to a certain Higgs effect in gauged AdS_5 supergravity since it shares a supermultiplet with the chiral R-symmetry anomaly on the field theory side. We first analyze on gravity side which local symmetry corresponding to the scale symmetry on field theory side becomes broken in K-S solution. Obviously, this local symmetry is not directly related to the isometry symmetry of internal manifold $T^{1,1}$. It should lie in the diffeomorphism symmetry of AdS_5 space. The argument for this statement is implied from Ref. [34], where it was shown that near the AdS_5 boundary the diffeomorphism symmetry of AdS_5 metric decomposes into a combination of four-dimensional diffeomorphism symmetry and Weyl symmetry. The AdS/CFT correspondence at supergravity level tells us that the four-dimensional diffeomorphism- and Weyl symmetries are equivalent to the conservation and tracelessness of energy-momentum

tensor in the dual four-dimensional supersymmetric gauge theory [35]. This suggests that we should observe how the diffeomorphism symmetry of AdS_5 space is spoiled by fractional branes to look for the dual of scale anomaly.

As discussing chiral anomaly, we first observe the case without fractional branes. Since we care about how the graviton is affected by the breaking of diffeormorphism symmetry of AdS_5 space, so only the Einstein-Hilbert- and the self-dual five-form terms in type IIB supergravity are selected out,

$$S_{\rm IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}xE\left(R - \frac{1}{4\times 5!}F_{(5)MNPQR}F_{(5)}{}^{MNPQR}\right) + \cdots . \tag{5.21}$$

Making the expansion

$$\begin{aligned}
G_{MN} &= G^{(0)}_{MN} + h_{MN},\\
G^{MN} &= G^{(0)MN} - h^{MN} + h^{MP}h_P{}^N + \mathcal{O}(h^3),\\
\sqrt{-G} &= \sqrt{-G_0}\left[1 + \frac{1}{2}h + \frac{1}{4}\left(\frac{1}{2}h^2 - h^{MN}h_{MN}\right)\right] + \mathcal{O}(h^3),\\
R^M{}_{NPQ} &= R^{(0)M}{}_{NPQ} + \frac{1}{2}\left[\nabla_P,\nabla_Q\right]h^M{}_N + \frac{1}{2}\left(\nabla_P\nabla_N h^M{}_Q - \nabla_Q\nabla_N h^M{}_P\right)\\
&\quad -\frac{1}{2}\left(\nabla_P\nabla^M h_{NQ} - \nabla_Q\nabla^M h_{NP}\right) - \frac{1}{2}h^{MR}\left[\nabla_P,\nabla_Q\right]h_{RN}\\
&\quad -\frac{1}{2}h^{MR}\left(\nabla_P\nabla_N h_{RQ} - \nabla_Q\nabla_N h_{RP}\right) + \frac{1}{2}h^{MR}\left(\nabla_P\nabla_R h_{NQ} - \nabla_Q\nabla_R h_{NP}\right)\\
&\quad +\frac{1}{4}\left(\nabla^R h^M{}_P - \nabla_P h^{MR} - \nabla^M h^R{}_P\right)\left(\nabla_Q h_{RN} + \nabla_N h_{RQ} - \nabla_R h_{NQ}\right)\\
&\quad -\frac{1}{4}\left(\nabla^R h^M{}_Q - \nabla_Q h^{MR} - \nabla^M h^R{}_Q\right)\left(\nabla_P h_{RN} + \nabla_N h_{RP} - \nabla_R h_{NP}\right)\\
&\quad +\mathcal{O}(h^3),
\end{aligned} \tag{5.22}$$

we obtain the quadratic action for the graviton of type IIB supergravity in the $AdS_5\times T^{1,1}$ background,

$$\begin{aligned}
S_{\rm Q.G.} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G^{(0)}}\left[-\frac{1}{4}\nabla^P h^{MN}\nabla_P h_{MN} + \frac{1}{2}\nabla^P h^{MN}\nabla_M h_{PN}\right.\\
&\quad +\frac{1}{4}\nabla^M h\left(\nabla_M h - \frac{1}{2}\nabla^N h_{MN}\right)\\
&\quad +\left(R^{(0)} - \frac{1}{4\times 5!}F^{(0)MNPQR}_{(5)}F^{(0)}_{(5)MNPQR}\right)\frac{1}{4}\left(\frac{1}{2}h^2 - h^{ST}h_{ST}\right)\\
&\quad -\frac{1}{4\times 5!}\left(10h^{M'[M}h^{NN'}F^{(0)}_{(5)MNPQR}F^{(0)}_{(5)M'N'}{}^{PQR]}\right.\\
&\quad \left.\left. -\frac{5}{2}h\,h^{M'[M}F^{(0)}_{(5)MNPQR}F^{(0)}_{(5)M'}{}^{NPQR]} + 5h^{M'M''}h_{M''}{}^{[M}F^{(0)}_{(5)MNPQR}F^{(0)}_{(5)M'}{}^{NPQR}\right)\right]\\
&= \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G^{(0)}}\left[-\frac{1}{4}\nabla^P h^{MN}\nabla_P h_{MN} + \frac{1}{2}\nabla^P h^{MN}\nabla_M h_{PN}\right.\\
&\quad +\frac{1}{4}\nabla^M h\left(\nabla_M h - \frac{1}{2}\nabla^N h_{MN}\right) + \frac{1}{2}R_{MPNQ}h^{MN}h^{PQ} - \frac{1}{2}h^{MN}h_N^P R_{PM}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{27}{32}\frac{\pi\alpha'^2 N}{h^2(r)r^5}\left(h^{MN}h_{MN}-\frac{1}{2}h^2\right)\Big]\\
&=\frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G^{(0)}}\left[-\frac{1}{4}\nabla^P h^{MN}\nabla_P h_{MN}+\frac{1}{2}\nabla_M h^{MN}\nabla^P h_{PN}\right.\\
&\left.+\frac{1}{4}\nabla^M h\left(\nabla_M h-\frac{1}{2}\nabla^N h_{MN}\right)-\frac{9}{32}\frac{\pi\alpha'^2 N}{h^2(r)r^5}\left(h^{MN}h_{MN}+\frac{1}{2}h^2\right)\right]. \qquad (5.23)
\end{aligned}
$$

The use of following identity is made in deriving above quadratic action,

$$
[\nabla_M,\nabla_P]\,h_N^{\ P}=R_{MPN}^{\ \ \ \ \ Q}h_Q^{\ P}-R_{MP}h_N^{\ P}. \qquad (5.24)
$$

We then perform the compactification of the quadratic action (5.23) on $T^{1,1}$. With gauge-fixing conditions $D^\xi h_{\xi\alpha}=D^\xi h_{(\xi\varepsilon)}=0$, the expansion of $h_{MN}(x,y)$ in terms of the $U_{\rm H}(1)$-harmonics on $T^{1,1}$ is the following [31],

$$
\begin{aligned}
h_{MN}(x,y) &= \left(h_{\alpha\beta}(x,y),h_{\alpha\xi}(x,y),h_{\xi\varepsilon}(x,y)\right),\\
h_{\alpha\beta}(x,y) &= \sum_{j,l,r} h_{\alpha\beta}^{(j,l,r)}(x)Y_0^{(j,l,r)}(y),\\
h_{\alpha\xi}(x,y) &= \sum_{j,l,r} A_\alpha^{(j,l,r)}(x)Y_\xi^{(j,l,r)}(y)=\sum_{i=1}^{5}\sum_{j,l,r_i}A_\alpha^{(j,l,r_i)}(x)Y_{q_i}^{(j,l,r_i)}(y),\\
h_{(\xi\varepsilon)}(x,y) &\equiv h_{\xi\varepsilon}-\frac{1}{5}g_{\xi\varepsilon}h^\tau{}_\tau=\sum_{j,l,r}\varphi^{(j,l,r)}(x)Y_{(\xi\varepsilon)}^{(j,l,r)}(y)\\
&= \sum_{i=1}^{10}\sum_{j,l,r_i}\varphi^{(j,l,r_i)}(x)Y_{q_i}^{(j,l,r_i)}(y),\\
h^\xi{}_\xi(x,y) &= \sum_{j,l,r}\pi^{(j,l,r)}(x)Y_0^{(j,l,r)}(y), \qquad (5.25)
\end{aligned}
$$

where D_ξ is the $SO(5)$ covariant derivative on $T^{1,1}$. The compactification on $T^{1,1}$ formally leads to the quadratic action for the graviton in $\mathcal{N}=2$ $U(1)$ gauged AdS_5 supergravity,

$$
\begin{aligned}
S_{\rm q.g.} = \frac{1}{2k_5^2}\int d^5x\sqrt{-g^{(0)}}&\left[-\frac{1}{4}\nabla^\gamma h^{\alpha\beta}\nabla_\gamma h_{\alpha\beta}+\frac{1}{2}\nabla^\gamma h^{\alpha\beta}\nabla_\alpha h_{\gamma\beta}\right.\\
&\left.+\frac{1}{4}\nabla^\alpha h\left(\nabla_\alpha h-\frac{1}{2}\nabla^\beta h_{\alpha\beta}\right)-\frac{1}{L^2}\left(h^{\alpha\beta}h_{\alpha\beta}+\frac{1}{2}h^2\right)+\cdots\right]. \qquad (5.26)
\end{aligned}
$$

The graviton $h_{\alpha\beta}(x)$ is the fluctuation around the metric of AdS_5 space (5.12),

$$
ds_5^2=g_{\alpha\beta}dx^\alpha dx^\beta=\left[g_{\alpha\beta}^{(0)}+h_{\alpha\beta}(x)\right]dx^\alpha dx^\beta. \qquad (5.27)
$$

The $SO(2,4)$ diffeomorphism invariance means the following infinitesimal gauge symmetry for the graviton in AdS_5 space,

$$
\delta h_{\alpha\beta}=\nabla_\alpha^{(0)}\xi_\beta+\nabla_\beta^{(0)}\xi_\alpha, \qquad (5.28)
$$

where the covariant derivative $\nabla_\alpha^{(0)}$ is defined with respect to the metric $g_{\mu\nu}^{(0)}$ of AdS_5 space. Near the AdS_5 boundary, $\xi_\alpha=(\xi_\mu,\xi_r)$, the above bulk diffeomorphism transformation

(5.28) decomposes into four-dimensional diffeormorphism- and Weyl transformations, ξ_μ and ξ_r playing the roles of transformation parameters, respectively [34]. The action (5.26) determines the equation of motion for the graviton in AdS_5 space [36],

$$\begin{aligned} E_{\alpha\beta} \equiv & \frac{1}{2}\left(\nabla^\gamma\nabla_\gamma h_{\alpha\beta} - \nabla_\alpha\nabla_\gamma h^\gamma{}_\beta - \nabla_\beta\nabla_\gamma h^\lambda{}_\alpha + \nabla_\alpha\nabla_\beta h\right) \\ & + \frac{1}{2}g^{(0)}_{\alpha\beta}\left(\nabla_\gamma\nabla_\delta h^{\gamma\delta} - \nabla^\gamma\nabla_\gamma h\right) - \frac{2}{L^2}\left(h_{\alpha\beta} + \frac{1}{2}g^{(0)}_{\alpha\beta}h\right) = 0. \end{aligned} \tag{5.29}$$

Due to the gauge symmetry (5.28), $h_{\alpha\beta}$ contain non-physical modes and the physical ones should be the traceless and transverse part of $h_{\alpha\beta}$. Eq. (5.29) leads to the identity, $\nabla^{(0)}_\beta E^{\alpha\beta} = 0$. This identity and the gauge-fixing condition $\nabla^{(0)}_\alpha h^{\alpha\beta} = 0$ fixes the physical degrees of freedom of the AdS_5 graviton.

When fractional $D3$-branes switch on, the $AdS_5 \times T^{1,1}$ background get deformed by R-R- and NS-NS three-form fluxes carried by fractional branes. The isometric symmetry of the deformed $AdS_5 \times T^{1,1}$ reduces. So the compactified theory obtained from the action (5.23) in the deformed $AdS_5 \times T^{1,1}$ background should suffer from the spontaneous symmetry breaking from $SO(2,4)$ to $SO(1,4)$ and present the consequent Higgs effect. In the following we demonstrate how this Higgs mechanism occurs.

The action of type IIB supergravity composed of the self-dual five-form $F_{(5)}$, the NS-NS two-form $B_{(2)}$ and the Einstein-Hilbert part is

$$\begin{aligned} S_{\rm IIB} = & \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-\overline{G}}\left(\overline{R} - \frac{1}{2\times 3!}H_{(3)MNP}H_{(3)}^{MNP}\right. \\ & \left. - \frac{1}{4\times 5!}F_{(5)MNPQR}F_{(5)}^{MNPQR}\right). \end{aligned} \tag{5.30}$$

Note that for simplicity of discussion we do not consider terms relevant to the R-R three-form. The Einstein equation from this classical action is

$$\begin{aligned} \overline{R}_{MN} = & \frac{1}{96}F_{(5)MPQRS}F_{(5)N}{}^{PQRS} \\ & + \frac{1}{4}\left(H_{(3)MPQ}H_{(3)N}{}^{PQ} - \frac{1}{12}G_{MN}H_{(3)PQR}H^{(3)PQR}\right). \end{aligned} \tag{5.31}$$

This equation shows that without three-form field strength $H_{(3)PQR}$, the self-dual five-form flux leads to the three-brane solution (3.1) whose near-horizon limit is $AdS_5 \times T^{1,1}$ if the flat limit is $M^4 \times \mathcal{C}_6$ [37], while the presence of NS-NS three-form flux makes a deformation on $ASdS_5 \times T^{1,1}$ and leads to a less symmetric vacuum background. According to the basic idea of the Higgs mechanism, we should make the vacuum configuration symmetric by performing a gauge transformation which re-parametrises the field variable. This can be done by shifting the Ricci curvature and absorbing the three-form flux contribution into it,

$$R \equiv \overline{R} - \frac{1}{2\times 3!}H_{(3)MNP}H_{(3)}{}^{MNP}. \tag{5.32}$$

Consequently, the above Einstein equation becomes the one with only five-form fluxes as source,

$$\begin{aligned} R_{MN} &\equiv \overline{R}_{MN} - \frac{1}{4}\left(H_{(3)MPQ}H_{(3)N}{}^{PQ} - \frac{1}{12}G_{MN}H_{(3)PQR}H_{(3)}^{PQR}\right) \\ &= \frac{1}{96}F_{(5)MPQRS}F_{(5)N}{}^{PQRS}. \end{aligned} \tag{5.33}$$

It gives the three-brane solution (3.1), the symmetric vacuum background. Eq. (5.33) implies that there exists following connection between deformed and undeformed Riemannian curvatures,

$$\begin{aligned} R_{KMLN} = \overline{R}_{KMLN} - \frac{1}{4}\Big[&H_{(3)KMQ}H_{(3)LN}{}^{Q} \\ &- \frac{1}{12\times 9}\left(G_{KL}G_{MN} - G_{KN}G_{ML}\right)H_{(3)PQR}H_{(3)}^{PQR}\Big]. \end{aligned} \tag{5.34}$$

We expand the gravitational field around the symmetric $AdS_5 \times T^{1,1}$ vacuum configuration. However, just like the Higgs phenomenon in gauge theory, h_{MN} must undergo a gauge transformation, which can make h_{MN} pick up a longitudinal component and become massive. This is, the following operation should be made on background metrics and graviton fields,

$$G_{MN} = \overline{G}^{(0)}_{MN} + h_{MN} = G^{(0)}_{MN} + \overline{h}_{MN}, \tag{5.35}$$

where $G^{(0)}_{MN}$ and $\overline{G}^{(0)}_{MN}$ denote the symmetric and deformed $AdS_5 \times T^{1,1}$ metrics, respectively, and $\overline{h}_{MN}$ and h_{MN} are graviton fluctuations around these two space-time backgrounds, respectively.

The above discussion is a qualitative analysis in ten dimensions. In the following we use the K-T solution to find the explicit shift and make the Higgs phenomenon more clear. According to the K-T solution (3.6),

$$\begin{aligned} ds^2_{\mathrm{d-AdS_5}} &= \overline{g}^{(0)}_{\alpha\beta}dx^\alpha dx^\beta \approx \frac{r^2}{L^2}\left[1 - A(r)\right]\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}\left[1 + A(r)\right]dr^2, \\ A(r) &= \frac{3}{4\pi}\frac{M^2}{N}g_s\left(\frac{1}{4} + \ln\frac{r}{r_0}\right), \end{aligned} \tag{5.36}$$

This deformed AdS_5 background shows that it is the logarithmic dependence on the radial coordinate that breaks the Weyl symmetry corresponding to scale symmetry on field theory side. Consequently, the $SO(2,4)$ isometry symmetry of AdS_5 space breaks to $SO(1,4)$.

We restore the deformed AdS_5 vacuum background back to AdS_5. The $(\alpha\beta)$ component of Eqs. (5.33) and the explicit form of NS-NS two-form $B_{(2)}$ given in (3.6) determine that there should exist

$$\overline{g}^{(0)}_{\alpha\beta} = g^{(0)}_{\alpha\beta} - \nabla_\alpha B_\beta - \nabla_\beta B_\alpha. \tag{5.37}$$

In above equation, B_α is a vector field originating from NS-NS two-form field $B_{(2)}$ in the K-S solution,

$$B_\alpha = \partial_\alpha\left[G\int_{S^2} B_{(2)}\right], \tag{5.38}$$

where G is a certain function in ten dimensions. Since the deformed AdS_5 vacuum configuration transforms as

$$\delta\overline{g}^{(0)}_{\alpha\beta} = \nabla^{(0)}_{\alpha}\xi_{\beta} + \nabla^{(0)}_{\beta}\xi_{\alpha}, \tag{5.39}$$

the shifted background metric $g^{(0)}_{\alpha\beta}$ is invariant under the diffeomorphism transformation,

$$\delta x^{\alpha} = -\xi^{\alpha}, \quad \delta B_{\alpha} = -\xi_{\alpha}, \tag{5.40}$$

Therefore, the AdS_5 space-time background is re-gained and $SO(2,4)$ symmetry is recovered. A straightforward argument for this vacuum configuration restoration is that B_α provides a cancelation to the non-symmetric part since B_α also has $\ln(r/r_0)$ dependence.

From Eqs. (5.35) and (5.37), the graviton in the symmetric AdS_5 background should be

$$h_{\alpha\beta} = \overline{h}_{\alpha\beta} + \nabla^{(0)}_{\alpha}B_{\beta} + \nabla^{(0)}_{\beta}B_{\alpha}. \tag{5.41}$$

This shows that the graviton undergoes a gauge transformation from which it picks up a longitudinal component and B_α plays the role of a Goldstone vector field [38].

Based on above analysis, we expand the classical action (5.30) to the second order of graviton $\overline{h}_{MN}$ around the symmetric AdS_5 background. Further, all of other fields such as $B_{(2)}$ must be replaced by their $SO(2,4)$ transformed versions if they lie in certain non-trivial representations of $SO(2,4)$. The quadratic action of type IIB supergravity in deformed $AdS_5 \times T^{1,1}$ background can be recast into the one around the symmetric $AdS_5 \times T^{1,1}$ vacuum configuration,

$$\begin{aligned}
\overline{S}_{\text{Q.G.}} &= \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G^{(0)}}\left\{-\frac{1}{4}\nabla^P\overline{h}^{MN}\nabla_P\overline{h}_{MN} + \frac{1}{2}\nabla^P\overline{h}^{MN}\nabla_M\overline{h}_{PN}\right.\\
&\quad + \frac{1}{4}\nabla^M\overline{h}\left(\nabla_M\overline{h} - \frac{1}{2}\nabla^N\overline{h}_{MN}\right) + \frac{27}{32}\frac{\pi\alpha'^2N}{\overline{h}^2(r)r^5}\left(\overline{h}^{MN}\overline{h}_{MN} - \frac{1}{2}h^2\right)\\
&\quad \left. + \frac{1}{2}\left[R^{(0)}_{MPNQ} - \frac{1}{90}\left(G^{(0)}_{MN}G^{(0)}_{PQ} - G^{(0)}_{MQ}G^{(0)}_{NP}\right)\frac{1}{2\times 3!}H^{(0)}_{(3)RST}H^{(0)RST}_{(3)}\right]\overline{h}^{MN}\overline{h}^{PQ}\right\}\\
&= \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G^{(0)}}\left[-\frac{1}{4}\nabla^P\overline{h}^{MN}\nabla_P\overline{h}_{MN} + \frac{1}{2}\nabla_M\overline{h}^{MN}\nabla^P\overline{h}_{PN}\right.\\
&\quad +\frac{1}{4}\nabla^M\overline{h}\left(\nabla_M\overline{h} - \frac{1}{2}\nabla^N\overline{h}_{MN}\right) - \frac{9}{32}\frac{\pi\alpha'^2N}{h^2(r)r^5}\left(\overline{h}^{MN}\overline{h}_{MN} + \frac{1}{2}\overline{h}^2\right)\\
&\quad \left. -\frac{1}{4}\times\frac{1}{2\times 45}\left(\frac{3g_sM\alpha'}{4}\right)^2\frac{72}{r^6h^{3/2}(r)}\left(\overline{h}^{MN}\overline{h}_{MN} - \overline{h}^2\right)\right].
\end{aligned} \tag{5.42}$$

The last term is the celebrated Pauli-Fierz mass term for type IIB graviton [39].

Further, performing compactification on $T^{1,1}$ and taking into account only zero modes in the K-K spectrum, we obtain the massive AdS_5 graviton due to the spontaneous breaking of local $SO(2,4)$ symmetry to $SO(1,4)$,

$$\overline{S}_{\text{q.g.}} = \frac{1}{2k_5^2}\int d^5x\sqrt{-g^{(0)}}\left[-\frac{1}{4}\nabla^{\gamma}\overline{h}^{\alpha\beta}\nabla_{\gamma}\overline{h}_{\alpha\beta} + \frac{1}{2}\nabla^{\gamma}\overline{h}^{\alpha\beta}\nabla_{\alpha}\overline{h}_{\gamma\beta}\right.$$

$$+\frac{1}{4}\nabla^\alpha\overline{h}\left(\nabla_\alpha\overline{h}-\frac{1}{2}\nabla^\beta\overline{h}_{\alpha\beta}\right)-\frac{1}{L^2}\left(\overline{h}^{\alpha\beta}\overline{h}_{\alpha\beta}+\frac{1}{2}\overline{h}^2\right)$$
$$\left.-\frac{1}{4}m^2\left(\overline{h}^{\alpha\beta}\overline{h}_{\alpha\beta}-\overline{h}^2\right)\right]. \tag{5.43}$$

The mass m can be evaluated by integrating over the internal manifold $T^{1,1}$.

5.5 Dual of γ-trace Anomaly $\gamma_\mu s^\mu$ for Supersymmetry Current

Finally we tackle the last member in the superconformal anomaly multiplet, the dual of γ-trace anomaly $\gamma^\mu s_\mu$ of supersymmetry current s_μ. As discussed in Sect. 4, the breaking of super-Weyl symmetry by fractional branes is revealed by the Killing spinor equation obtained from the supersymmetry transformation on the graviton Ψ_M and dilatino Λ in the K-S solution background. The Killing spinor equations in the symmetric and deformed $AdS_5\times T^{1,1}$ background are listed in Eq. (4.10) and Eq. (4.14), respectively. We employ the same idea as looking for the dual of scale anomaly to uncover the dual of γ-trace anomaly. That is, we should shift Λ and Ψ_M so that the Killing spinor equation (4.14) in deformed $AdS_5\times T^{1,1}$ background recovers the form of Eq. (4.10), the Killing spinor equation in symmetric $AdS_5\times T^{1,1}$ background.

A comparison between (4.10) and (4.14) shows that we should introduce a complex right-handed Weyl spinor Υ and make shift,

$$\Lambda'\equiv\Lambda-4i\Upsilon,\quad \Psi'_M\equiv\Psi_M-\Gamma_M\Upsilon. \tag{5.44}$$

Using the property of Γ-matrix in ten dimensions,

$$\begin{aligned}\Gamma_{M_1M_2\cdots M_n}&=\Gamma_{[M_1}\Gamma_{M_2}\cdots\Gamma_{M_n]}=\frac{1}{n!}\sum_P(-1)^{\delta_P}\Gamma_{a_{P(1)}}\Gamma_{a_{P(2)}}\cdots\Gamma_{a_{P(n)}},\\ \Gamma_{M_1M_2\cdots M_nN}&=\Gamma_{M_1M_2\cdots M_n}\Gamma_N-n\Gamma_{[M_1M_2\cdots M_{n-1}}G_{M_n]N},\\ \Gamma_{NM_1M_2\cdots M_n}&=\Gamma_N\Gamma_{M_1M_2\cdots M_n}-nG_{N[M_1}\Gamma_{M_1M_2\cdots M_n]},\end{aligned} \tag{5.45}$$

we find that Υ should be assigned to the supersymmetry transformation

$$\delta\Upsilon=-\frac{1}{96}\left(F_{(3)MNP}+iH_{(3)MNP}\right)\Gamma^{MNP}\epsilon \tag{5.46}$$

so that (4.10) can be reproduced from (4.14) (up to the linear term in fermionic fields and to the first order in the gravitational coupling κ_{10}). After the compactification, Υ will yield a Goldstone fermion needed for a super-Higgs mechanism, which should arise since the NS-NS- and R-R three-form fluxes breaks one-half of local supersymmetries. Eq. (5.46) implies that the supersymmetric transformation for the compactified Υ is proportional to the transformation parameter with the proportionality coefficient given by the three-form fluxes passing through S^3. This is a typical feature of the Goldstone fermion [40].

We focus on the quadratic gravitino action of type IIB supergravity,

$$\begin{aligned}S_{\rm gravitino}=\frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G}&\left[-\frac{i}{2}\overline{\Psi}_M\Gamma^{MNP}D_N\Psi_P\right.\\ &\left.-\frac{1}{8\times 5!}\overline{\Psi}_M\Gamma^{MNP}\left(\Gamma^{UVWXY}F_{(5)UVWXY}\right)\Gamma_N\Psi_P+\cdots\right].\end{aligned} \tag{5.47}$$

From Eq. (4.2), when the fractional D3-branes are absent, the space-time background is $AdS_5 \times T^{1,1}$ with the self-dual five-form field strength,

$$F_{x_0x_1x_2x_3r} = \frac{r^3}{g_s L^4}, \quad F_{g_1g_2g_3g_4g_5} = \frac{L^4}{27 g_s}. \tag{5.48}$$

In the following we perform the compactification of fermionic action (5.47) on $T^{1,1}$. With the Γ-matrix representations listed in Appendix, Ψ_M, Λ and the supersymmetry transformation parameter ϵ decompose as the following,

$$\begin{aligned}
\Psi_\alpha(x,y) &= \widehat{\psi}_\alpha(x,y) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \widehat{\psi}_\alpha(x,y) \\ 0 \end{pmatrix}, \\
\Psi_\xi(x,y) &= \widehat{\psi}_\xi(x,y) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \widehat{\psi}_\xi(x,y) \\ 0 \end{pmatrix}, \\
\Lambda(x,y) &= \widehat{\Lambda}(x,y) \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \widehat{\Lambda}(x,y) \end{pmatrix}, \\
\epsilon(x,y) &= \widehat{\epsilon}(x,y) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \widehat{\epsilon}(x,y) \\ 0 \end{pmatrix}, \\
\Upsilon(x,y) &= \widehat{\chi}(x,y) \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \widehat{\chi}(x,y) \end{pmatrix},
\end{aligned} \tag{5.49}$$

where $\widehat{\psi}_\alpha$, $\widehat{\psi}_\xi$, $\widehat{\Lambda}$, $\widehat{\epsilon}$ and $\widehat{\chi}$ are the 16-component Weyl spinor in ten dimensions. We then expand these field functions and the supersymmetry transformation parameter in the four-component $SO(5)$ spinor harmonics, which further decompose into one-dimensional $U_{\rm H}(1)$-harmonics on $T^{1,1}$ [31], that is,

$$\widehat{\psi}_\alpha(x,y) = \sum_{j,l,r} \psi_\alpha^{(j,l,r)}(x)\Xi^{(j,l,r)}(y) = \sum_{j,l,r} \begin{pmatrix} \psi_\alpha^{(j,l,r-1)}(x)Y_0^{(j,l,r-1)}(y) \\ \psi_\alpha^{(j,l,r+1)}(x)Y_0^{(j,l,r+1)}(y) \\ \psi_\alpha^{(j,l,r)}(x)Y_{-1}^{(j,l,r)}(y) \\ \psi_\alpha^{(j,l,r)}(x)Y_{+1}^{(j,l,r)}(y) \end{pmatrix}. \tag{5.50}$$

$\widehat{\psi}_\xi(x,y)$, $\widehat{\Lambda}(x,y)$, $\widehat{\epsilon}(x,y)$ and $\widehat{\chi}(x,y)$ admit the same expansion in terms of the $U_{\rm H}(1)$-harmonics on $T^{1,1}$. Note that $\psi_\alpha^{(j,l,r)}(x)$, $\Lambda^{(j,l,r)}(x)$, and $\epsilon^{(j,l,r)}(x)$ are four-component spinors on AdS_5; $\Xi^{(j,l,r)}(y)$ are the four-component $SO(5)$ spinor harmonics on $T^{1,1}$, and $Y_q^{(j,l,r)}(y)$ are $U_{\rm H}(1)$-harmonics carrying $U_{\rm H}(1)$-charge q.

We substitute the chiral decomposition (5.49) and the $U_{\rm H}(1)$-harmonic expansions for Ψ_α and Λ into the quadratic action (5.47) and consider only zero modes of their kinetic operators defined on $T^{1,1}$. Further, integrating over $T^{1,1}$ we obtain the quadratic action for the graviton of the gauged AdS_5 supergravity,

$$S_{\rm gravitino} = \frac{1}{2\kappa_5^2}\int d^{10}x\, e\left(-\frac{i}{2}\overline{\psi}^i_\alpha\gamma^{\alpha\beta\gamma}D_\beta\psi_{i\gamma} + \frac{3}{4L}\overline{\psi}^i_\alpha\gamma^{\alpha\beta}\psi_{i\beta} + \cdots\right). \tag{5.51}$$

The second term in Eq. (5.51) is required by supersymmetry to accompany the cosmological term in AdS_5 space [41]; ψ^i_α is the $SU(2)$ symplectic Majorana spinor and the index $i = 1, 2$

labels two gravitini with $U(1)$-charge $r=\pm 1$, which arise automatically when performing compactification on $T^{1,1}$ [31].

On the other hand, the fractional $D3$ branes deform the $AdS_5\times T^{1,1}$ background. The gravitino action (5.47) can be expressed in terms of either Ψ_M, Λ in deformed $AdS_5\times T^{1,1}$ background or the shifted fields Ψ'_M and Λ' in the symmetric $AdS_5\times T^{1,1}$ background. We start from the second term of (5.47) in the deformed $AdS_5\times T^{1,1}$ background and re-write it in terms of shifted fermionic field variables (5.44) in the $AdS_5\times T^{1,1}$ background. Up to the leading order of κ_{10} we have

$$
\begin{aligned}
\overline{S}_{\text{gravitino}} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-\overline{G}}\left[-\frac{1}{8\times 5!}\overline{\Psi}_M\Gamma^{MNP}\left(\Gamma^{UVWXY}\overline{\widetilde{F}}_{(5)UVWXY}\right)\Gamma_N\Psi_P\right]\\
&= \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-G}\left(-\frac{1}{8\times 5!}\right)\left(\overline{\Psi}'_M+\overline{\Upsilon}\Gamma_M\right)\Gamma^{MNP}\left(\Gamma^{(5)}\cdot\widetilde{F}_{(5)}\right)\Gamma_N\left(\Psi'_P-\Gamma_P\Upsilon\right)\\
&= \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-G}\left(-\frac{1}{8\times 5!}\right)\left(\overline{\Psi}'_M\Gamma^{MNP}+8\overline{\Upsilon}\Gamma^{NP}\right)\left(\Gamma^{(5)}\cdot\widetilde{F}_{(5)}\right)\left(\Gamma_N\Psi'_P-\Gamma_{NP}\Upsilon\right)\\
&= -\frac{1}{2\kappa_{10}^2}\frac{1}{8\times 5!}\int d^{10}x\,\sqrt{-G}\,\Big[\left(\overline{\Psi}'_\alpha\Gamma^{\alpha\beta\gamma}+\overline{\Psi}'_\xi\Gamma^\xi\Gamma^{\beta\gamma}+8\overline{\Upsilon}\Gamma^{\beta\gamma}\right)\\
&\quad\times\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}+\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\left(\Gamma_\beta\Psi'_\gamma-\Gamma_{\beta\gamma}\Upsilon\right)\\
&\quad+\left(\overline{\Psi}'_\alpha\Gamma^\alpha\Gamma^{\xi\varepsilon}+\overline{\Psi}'_\tau\Gamma^{\tau\xi\varepsilon}+8\overline{\Upsilon}\Gamma^{\xi\varepsilon}\right)\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}+\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\left(\Gamma_\xi\Psi'_\varepsilon-\Gamma_{\xi\varepsilon}\Upsilon\right)\\
&\quad+\left(\overline{\Psi}'_\alpha\Gamma^{\alpha\beta}\Gamma^\xi+\overline{\Psi}'_\varepsilon\Gamma^{\varepsilon\xi}\Gamma^\beta+8\overline{\Upsilon}\Gamma^\beta\Gamma^\xi\right)\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}+\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\left(\Gamma_\beta\Psi'_\xi-\Gamma_\beta\Gamma_\xi\Upsilon\right)\\
&\quad+\left(-\overline{\Psi}'_\alpha\Gamma^{\alpha\beta}\Gamma^\xi+\overline{\Psi}'_\varepsilon\Gamma^{\varepsilon\xi}\Gamma^\beta+8\overline{\Upsilon}\Gamma^\beta\Gamma^\xi\right)\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}+\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\left(\Gamma_\xi\Psi'_\beta-\Gamma_\xi\Gamma_\beta\Upsilon\right)\Big]\\
&= -\frac{1}{2\kappa_{10}^2}\frac{1}{5!}\int d^{10}x\,\sqrt{-G}\,\Big[\frac{1}{4}\overline{\Psi}'_\alpha\Gamma^{\alpha\beta}\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}-\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}\right)\Psi'_\beta\\
&\quad+\overline{\Psi}'_\alpha\Gamma^\alpha\Gamma^\xi\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}-\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\Psi'_\xi\\
&\quad+\overline{\Upsilon}\Gamma^\alpha\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}-\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}\right)\Psi'_\alpha-\overline{\Psi}'_\alpha\Gamma^\alpha\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}+\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\Upsilon\\
&\quad-\overline{\Psi}'_\xi\Gamma^\xi\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}+\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\Upsilon+\overline{\Upsilon}\Gamma^\xi\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}-\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\Psi'_\xi\\
&\quad-2\times 5\times 5\overline{\Upsilon}\,\Gamma^\alpha\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}+\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\Upsilon\\
&\quad-\frac{1}{4}\,\overline{\Psi}'_\xi\Gamma^{\xi\varepsilon}\left(\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5}-\Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\Psi'_\varepsilon\Big],
\end{aligned}
\tag{5.52}
$$

where the following operation is performed,

$$
\begin{aligned}
\left[\Gamma^\alpha,\Gamma^{\alpha_1\cdots\alpha_5}\right]\widetilde{F}_{\alpha_1\cdots\alpha_5} &= \left(5g^{\alpha[\alpha_1}\Gamma^{\alpha_2\cdots\alpha_5]}-5\Gamma^{[\alpha_1\cdots\alpha_4}g^{\alpha_5]\alpha}\right)\widetilde{F}_{\alpha_1\cdots\alpha_5}\\
&= 5g^{\alpha\alpha_1}\Gamma_{\alpha_1}-5g^{\alpha\alpha_1}\Gamma_{\alpha_1}=0,\\
\left[\Gamma^\xi,\Gamma^{\xi_1\cdots\xi_5}\right]\widetilde{F}_{\xi_1\cdots\xi_5} &= \left[\Gamma^{\xi\varepsilon},\Gamma^{\xi_1\cdots\xi_5}\right]\widetilde{F}_{\xi_1\cdots\xi_5}=\left[\Gamma^{\alpha\beta},\Gamma^{\alpha_1\cdots\alpha_5}\right]\widetilde{F}_{\alpha_1\cdots\alpha_5}=0.
\end{aligned}
\tag{5.53}
$$

Further, the explicit Γ-matrix representation listed in (A.1) and (A.2) gives

$$
\Gamma^{\alpha_1\cdots\alpha_5}=-i\epsilon^{\alpha_1\cdots\alpha_5}1_4\otimes 1_4\otimes\sigma_1,\quad \Gamma^{\xi_1\cdots\xi_5}=-\epsilon^{\xi_1\cdots\xi_5}1_4\otimes 1_4\otimes\sigma_2,
$$

$$
\begin{aligned}
&\Gamma^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5} \pm \Gamma^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5} \\
&= -i1_4\otimes 1_4\otimes \begin{pmatrix} 0 & \epsilon^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5} \mp \epsilon^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5} \\ \epsilon^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5} \pm \epsilon^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5} & 0 \end{pmatrix}
\end{aligned} \tag{5.54}
$$

Substituting (5.54) and the Weyl spinor representations (5.49) into (5.52), we reduce $\overline{S}_{\text{gravitino}}$ to the form expressed in the ten-dimensional Weyl spinors,

$$
\begin{aligned}
\overline{S}_{\text{gravitino}} &= \frac{1}{2\kappa_{10}^2}\frac{i}{5!}\int d^{10}x\sqrt{-G}\left[\left(\frac{1}{4}\overline{\widehat{\psi}}_\alpha\gamma^{\alpha\beta}\widehat{\psi}_\beta - 2\overline{\widehat{\psi}}_\alpha\gamma^\alpha\widehat{\chi} + 5\overline{\widehat{\chi}}\widehat{\chi}\right)\right. \\
&\quad\times\left.\left(\epsilon^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5} - \epsilon^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right)\right] \\
&= -\frac{i}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left[\frac{3}{L}\frac{1}{g_s}\left(1+\frac{3}{2\pi}\frac{g_sM^2}{N}\ln\frac{r}{r_0}\right)\right. \\
&\quad\times\left.\left(\frac{1}{4}\overline{\widehat{\psi}}_\alpha\gamma^{\alpha\beta}\widehat{\psi}_\beta - 2\overline{\widehat{\psi}}_\alpha\gamma^\alpha\widehat{\chi} + 5\overline{\widehat{\chi}}\widehat{\chi}\right)\right].
\end{aligned} \tag{5.55}
$$

In above derivation we have used (5.48),

$$
\begin{aligned}
&\frac{1}{5!}\left(\epsilon^{\alpha_1\cdots\alpha_5}\widetilde{F}_{\alpha_1\cdots\alpha_5} - \epsilon^{\xi_1\cdots\xi_5}\widetilde{F}_{\xi_1\cdots\xi_5}\right) \\
&= -(-g_{AdS_5})^{-1/2}F_{0123r} - (g_{T^{1,1}})^{-1/2}F_{g^1g^2g^3g^4g^5} \\
&= -3h^{-5/4}(r)\frac{27}{4}\frac{\pi\alpha'^2N_{\text{eff}}}{r^5} = -\frac{3}{L}\frac{1}{g_s}\left(1+\frac{3}{2\pi}\frac{g_sM^2}{N}\ln\frac{r}{r_0}\right).
\end{aligned} \tag{5.56}
$$

Eq. (5.55) presents the typical feature of super-Higgs effects in supergravity [40] with $\widehat{\chi}$ playing the role of a Goldstone fermion. Substituting the $U_{\rm H}(1)$-harmonic expansions (5.50) for $\widehat{\psi}_\alpha$ and $\widehat{\chi}$ into (5.55), taking into account only zero modes, and further integrating over the internal manifold $T^{1,1}$, we obtain

$$
S_{\text{gravitino}} = -\frac{i}{2\kappa_5^2}\int d^5x\sqrt{-g^{(0)}}\left(\frac{3}{4L}\frac{1}{g_s}+m\right)\left(\overline{\psi}^i_\alpha\gamma^{\alpha\beta}\psi_{i\beta} - 8\overline{\psi}^i_\alpha\gamma^\alpha\chi_i + 20\overline{\chi}^i\chi_i\right) \tag{5.57}
$$

Finally, with a shift $\psi^{i\prime}_\mu = \psi^i_\mu - \gamma_\mu\chi^i$, an elegant result appears,

$$
S_{\text{gravitino}} = -\frac{i}{2\kappa_5^2}\int d^5x\sqrt{-g^{(0)}}\left(\frac{3}{4L}\frac{1}{g_s}+m\right)\overline{\psi}^{i\prime}_\alpha\gamma^{\alpha\beta}\psi^{i\prime}_\beta. \tag{5.58}
$$

The first term with coefficient proportional to $1/(Lg_s)$ in (5.58) accompanies the cosmological constant term, which is required by the Poincaré supersymmetry [41]; the second one is the mass term for the gravitino in five-dimensional gauged supergravity. This mass is generated by super-Higgs mechanism and is proportional to fluxes M carried by fractional branes, which can be seen clearly from Eq. (5.55).

5.6 Goldstone Hypermultiplet in AdS_5 Space

We have shown that the superconformal anomaly multiplet of an $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ supersymmetric gauge theory in four dimensions is dual to the spontaneous breaking of local supersymmetry and the consequent super-Higgs mechanism in $\mathcal{N}=2$ $U(1)$

gauged AdS_5 supergravity, through which the $\mathcal{N}=2$ graviton supermultiplet $(h_{\alpha\beta},\psi^i_\alpha,A_\alpha)$ becomes massive. A crucial ingredient in implementing this super-Higgs mechanism is the Goldstone fields θ, B_α and Υ in ten dimensions, which are defined in Eqs. (5.17), (5.38), (5.44) and (5.46), respectively. Since the superconformal anomaly in an $\mathcal{N}=1$ four-dimensional supersymmetric gauge theory is an $\mathcal{N}=1$ chiral supermultiplet [17], so according to the AdS/CFT correspondence conjecture [1, 2, 3], θ, B_α and Υ should constitute a supermultiplet. Specifically, the holographic version on AdS/CFT correspondence at supergravity level [2, 3] requires that after the compactification on $T^{1,1}$ θ, B_α and Υ should form an $\mathcal{N}=2$ Goldstone supermultiplet (hypermultiplet) in AdS_5 space and its AdS_5 boundary value should be an $\mathcal{N}=1$ chiral supermultiplet in four dimensions. In the following we verify this fact.

As a first step, we start from the supersymmetric transformations for $B_{(2)MN}$ and $F_{(3)MNP}$ [25],

$$\begin{aligned}\delta G_{(3)MNP} &= 3\partial_{[M}\delta A_{(2)NP]} = \delta\left[F_{(3)MNP}+iH_{(3)MNP}\right]\\ &= 3\partial_{[M}\delta C_{(2)NP]}+3i\partial_{[M}\delta B_{(2)NP]}\\ &= 3\partial_{[M}\left(\overline{\epsilon}\Gamma_{NP]}\Lambda\right)-12i\partial_{[M}\left(\overline{\epsilon}^\star\Gamma_N\Psi_{P]}\right).\end{aligned}\qquad(5.59)$$

This shows that locally there should exist

$$\delta A_{(2)NP}=\delta C_{(2)NP}+i\delta B_{(2)NP}=\overline{\epsilon}\Gamma_{NP}\Lambda-2i\overline{\epsilon}^\star\left(\Gamma_N\Psi_P-\Gamma_P\Psi_N\right).\qquad(5.60)$$

When we restore the deformed $AdS_5\times T^{1,1}$ back to the symmetric $AdS_5\times T^{1,1}$ vacuum background, the fermionic fields Ψ_M and Λ are reparametrized as in (5.44), the supersymmetric transformation for shifted $A_{(2)NP}$ becomes

$$\begin{aligned}\delta\overline{A}_{(2)NP} &= \delta\overline{C}_{(2)NP}+i\delta\overline{B}_{(2)NP}\\ &= \overline{\epsilon}\Gamma_{NP}\left(\widehat{\lambda}-4i\Upsilon\right)-2i\overline{\epsilon}^\star\left[\Gamma_N\left(\Psi_P-\Gamma_P\Upsilon\right)-\Gamma_P\left(\Psi_N-\Gamma_P\Upsilon\right)\right]\\ &= \overline{\epsilon}\Gamma_{NP}\widehat{\lambda}-2i\overline{\epsilon}^\star\left(\Gamma_N\Psi_P-\Gamma_P\Psi_N\right)-4i\left(\overline{\epsilon}-\overline{\epsilon}^\star\right)\Gamma_{NP}\Upsilon.\end{aligned}\qquad(5.61)$$

This leads to the supersymmetric transformation involving the Goldstone spinor Υ,

$$\delta A^{(\Upsilon)}_{(2)NP}=-4i\left(\overline{\epsilon}-\overline{\epsilon}^\star\right)\Gamma_{NP}\Upsilon=4\left(\mathrm{Im}\,\overline{\epsilon}\right)\Gamma_{NP}\Upsilon,\qquad(5.62)$$

where we use $A^{(\Upsilon)}_{(2)NP}$ to represent the parts of shifted $B_{(2)MN}$ and $C_{(2)MN}$ whose supersymmetric transformations depend only on Υ.

We turn to the supersymmetric transformation (5.46) for Υ,

$$\begin{aligned}\delta\Upsilon &= -\frac{1}{96}\times 3\,\partial_{[M}A^{(\Upsilon)}_{(2)NP]}\Gamma^{MNP}\epsilon\\ &= -\frac{1}{96}\times 3\left(\partial_{[M}C^{(\Upsilon)}_{(2)NP]}+i\partial_{[M}B^{(\Upsilon)}_{(2)NP]}\right)\Gamma^{MNP}\epsilon\\ &= -\frac{1}{32}\Gamma^M\left(\partial_M A^{(\Upsilon)}_{(2)NP}\right)\Gamma^{NP}\epsilon,\end{aligned}\qquad(5.63)$$

where the following Γ-matrix relation is employed,

$$\Gamma^{MNP} = \Gamma^M \Gamma^{NP} - \left(G^{MN}\Gamma^P - G^{MP}\Gamma^N\right). \tag{5.64}$$

Eqs. (5.62) and (5.63) show that $A^{(\Upsilon)}_{(2)NP}$ and Υ constitute a supermultiplet in ten dimensions since their supersymmetric transformations form a closed algebra.

Further, from the definitions (5.17) and (5.41) on the Goldstone bosons,

$$\theta = F\int_{S^2} C_{(2)}, \quad B_\alpha = \partial_\alpha\left[G\int_{S^2} B_{(2)}\right] \equiv \partial_\alpha \omega, \tag{5.65}$$

we choose $F = G$ and define a new complex scalar field in ten dimensions,

$$\widehat{\phi} \equiv \theta + i\omega = F\left[\int_{S^2}\left(C_{(2)} + iB_{(2)}\right)\right] \tag{5.66}$$

It should be emphasized that the choice $F = G$ is always possible since in (5.17) and (5.41) F and G are introduced as arbitrary field functions in ten dimensions. Actually, the requirement that the Goldstone fields should constitute a supermultiplet imposes this choice. Furthermore, since in the K-S solution both $C_{(2)}$ and $B_{(2)}$ are proportional to $\omega_2 = \left(g^1 \wedge g^2 + g^3 \wedge g^4\right)/2$, the complex scalar field $\widehat{\phi}$ defined in (5.66) actually contains two complex (four real) scalar fields,

$$\widehat{\phi}^1 \sim \int_{S^2}\left(C_{(2)g_1g_2} + iB_{(2)g_1g_2}\right), \quad \widehat{\phi}^2 \sim \int_{S^2}\left(C_{(2)g_3g_4} + iB_{(2)g_3g_4}\right). \tag{5.67}$$

Consequently, the supersymmetric transformations (5.62) and (5.63) for the Goldstone multiplet $(A^{(\Upsilon)}_{(2)NP}, \Upsilon)$ take the following form,

$$\begin{aligned}
\delta\widehat{\phi}^1 &= 4\left(\mathrm{Im}\,\overline{\epsilon}\right)\Gamma_{g_1g_2}\widetilde{\Upsilon}, \quad \delta\widehat{\phi}^2 = 4\left(\mathrm{Im}\,\overline{\epsilon}\right)\Gamma_{g_3g_4}\widetilde{\Upsilon}, \\
\delta\widetilde{\Upsilon} &= -\frac{1}{32}\Gamma^M\left[\left(\partial_M\phi^1\right)\Gamma_{g_1g_2} + \left(\partial_M\phi^2\right)\Gamma_{g_3g_4}\right]\epsilon, \quad \widetilde{\Upsilon} \equiv \int_{S^2}\Upsilon.
\end{aligned} \tag{5.68}$$

Recall that Υ and its supersymmetric transformation (5.46) are introduced to counter the supersymmetric transformation in the K-S solution background so that the Killing spinor equation (4.14) can recover to the Killing spinor equation (4.10) in $AdS_5 \times T^{1,1}$ background. Therefore, the supersymmetry transformation parameter ϵ in (5.62), (5.63) and (5.68) should satisfy the constraint equations (4.17). Because we use the UV (large-τ) limit of the K-S solution, i.e., the K-T solution, Eq. (4.17) gives

$$\Gamma_{g_1g_2}\epsilon = -\Gamma_{g_3g_4}\epsilon = i\left(\frac{\sinh\tau}{\cosh\tau} - \frac{1}{\cosh\tau}\Gamma_{g_1g_3}\right)\epsilon\bigg|_{\tau\to\infty} = i\epsilon. \tag{5.69}$$

So the supersymmetric transformation (5.68) of the Goldstone multiplet reduces to an elegant form,

$$\begin{aligned}
\delta\widehat{\phi}^1 &= -4i\,\left(\mathrm{Im}\,\overline{\epsilon}\right)\widetilde{\Upsilon}, \quad \delta\widehat{\phi}^2 = 4i\,\left(\mathrm{Im}\,\overline{\epsilon}\right)\widetilde{\Upsilon}, \\
\delta\widetilde{\Upsilon} &= -\frac{i}{32}\Gamma^M\left(\partial_M\widehat{\phi}^1 - \partial_M\widehat{\phi}^2\right)\epsilon.
\end{aligned} \tag{5.70}$$

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Substituting the chiral decomposition of Υ and ϵ listed in (5.49) into (5.70) and using

$$\overline{\epsilon} = \epsilon^\dagger \Gamma_0 = \left(\widehat{\epsilon}^\dagger(x,y), 0\right) \gamma^0 \otimes 1_4 \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \left(0, \overline{\widehat{\epsilon}}(x,y)\right) \tag{5.71}$$

we obtain

$$\begin{aligned} \delta\widehat{\phi}^1(x,y) &= -4i\left(\mathrm{Im}\,\overline{\widehat{\epsilon}}\right)\widehat{\widehat{\chi}}(x,y), \quad \delta\widehat{\phi}^2(x,y) = 4i\left(\mathrm{Im}\,\overline{\widehat{\epsilon}}\right)\widehat{\widehat{\chi}}(x,y), \\ \delta\widehat{\widehat{\chi}}(x,y) &= -\frac{i}{32}\left[\gamma^\alpha\partial^x_\alpha\left(\widehat{\phi}^1 - \widehat{\phi}^2\right) - i\tau^\xi\partial^y_\xi\left(\widehat{\phi}^1 - \widehat{\phi}^2\right)\right]\widehat{\epsilon}(x,y). \end{aligned} \tag{5.72}$$

The following task is performing compactification on $T^{1,1}$ and reducing above supersymmetric transformations to AdS_5 space. We expand $\widehat{\phi}^p(x,y)$ $(p = 1,2)$, $\widehat{\widehat{\chi}}(x,y)$ and $\widehat{\epsilon}(x,y)$ in terms of the scalar- and spinor $U_H(1)$ harmonics as shown in (5.50), and substitute the expansions into Eq. (5.72). Taking into account only zero modes in the expansions and comparing both sides of Eq. (5.72), we obtain

$$\delta\phi^p = 4if^p_{ij}\overline{\eta}^i\chi^j, \qquad \delta\chi^i = -\frac{1}{32}if^{ij}_p\gamma^\alpha\partial_\alpha\phi^p\eta_j. \tag{5.73}$$

In (5.73), ϕ^p, χ^i and η^i are the scalar fields, fermionic fields and $\mathcal{N} = 2$ supersymmetry transformation parameters in five dimensions obtained from the compactification of $\widehat{\phi}^p$, $\widehat{\widehat{\chi}}$ and $\widehat{\epsilon}$, respectively; $i,j = 1,2$ are the indices labeling fermions with opposite $U(1)$-charges $r = \pm 1$, which arise naturally in performing the compactification of fermionic fields on $T^{1,1}$, $f^{ij}_p \equiv \delta^{ij}N^{jp}$ (p is not summed); N^{vA} are defined as the following: $N^{11} = N^{21} = 1$ and $N^{12} = N^{22} = -1$; f^p_{ij} is related to f^{ij}_p as the following, $f^p_{ij} \equiv \epsilon^{pq}\delta_{ik}\delta_{jl}f^{kl}_q$. Eq. (5.73) is exactly the supersymmetry transformation for $\mathcal{N} = 2$ hypermultiplet in five-dimensions [14].

We should further take the hypermultiplet (ϕ^p, χ^i) to the boundary of AdS_5 space. In principle, we can take a similar procedure as in Ref. [42], where on-shell fields in gauged AdS_{p+1} supergravity can reduce to the off-shell fields of p-dimensional conformal supergravity on AdS_{p+1} boundary. However, in the case at hand the equations of motion for ϕ^p and χ^i are not clear. We naively take them on the boundary of AdS_5 space and assume rudely that Eq. (5.73) should lead to the supersymmetry transformations for $\mathcal{N} = 1$ chiral supermultiplet in four dimensions. This feature corresponds to the superconformal anomaly in a four-dimensional $\mathcal{N} = 1$ supersymmetric gauge theory.

6. Summary

We have investigated the supergravity dual description to the superconformal anomaly of a four-dimensional supersymmetric gauge theory. We make use of two distinct properties of D-brane in type II superstring theory: On one hand, in the weak coupling case, it behaves as a dynamical and geometric object with open strings ending on it. Thus a supersymmetric gauge theory on the world-volume of a stack of coincident D-branes can be constructed by an appropriate construction on brane configuration, and hence all the quantum phenomena of a gauge theory can be extracted out from D-brane dynamics; On

the other hand, a D-brane is charged with respect to the R-R string states and hence a stack of D-branes modify the space-time background of type II superstring theory. This type of effect of D-branes make them behave as the brane solution to type II supergravity in a strongly coupled type II superstring theory. We specialize to $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ Yang-Mills theory with two matter fields in the bifundamental representations $(N+M,\overline{N})$ and $(\overline{N+M},N)$ of gauge groups and a quartic superpotential. The brane configuration producing this supersymmetric gauge theory consists of N bulk $D3$-branes and M fractional $D3$-branes in the singular target space-time $M^4\times\mathcal{C}_6$. The fractional branes are fixed at the apex of the conifold $\mathcal{C}^6$ whose base is the five-dimensional Einstein manifold $T^{1,1}\sim S^2\times S^3$. On the other hand, the space-time background arising from the brane configuration is the celebrated Klebanov-Strassler solution. It is a non-singular solution to type IIB supergravity and its UV limit can be considered a deformed $AdS_5\times T^{1,1}$. We start from the field theory result on superconformal anomaly and track its origin to the D-brane configuration. We realize that the fractional branes frozen are the origin for superconformal anomaly: When fractional branes are absent, the field theory is an $\mathcal{N}=1$ $SU(N)\times SU(N)$ supersymmetric gauge theory with two flavors in the bifundamental representation $(N,\overline{N})$ and $(\overline{N},N)$, which is a superconformal quantum gauge theory at the IR fixed point of its renormalization group flow; While when fractional branes switch on, the field theory becomes an $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ supersymmetric gauge theory with two flavors in the bifundmental representation $(N+M,\overline{N})$ and $(\overline{N+M},N)$, which ceases to be a superconformal theory. Then we go to the strongly coupled side of type II superstring and make use of the gravitational feature of D-branes. We find that the effect of the fractional $D3$-branes is to deform $AdS_5\times T^{1,1}$ space-time: Without fractional branes the near-horizon limit of three-brane solution yielded from above brane configuration is $AdS_5\times T^{1,1}$; While with the fractional branes present, the corresponding three-brane solution transits to the K-S solution. We choose the (UV limit of) K-S solution as a vacuum configuration for type IIB supergravity. Due to the geometric meaning of a gravity theory, the spontaneous compactification on the deformed $T^{1,1}$ takes place. Since in the Kaluza-Klein compactification, the isometry symmetry of the background space-time and the supersymmetry it preserves convert into local symmetries for the compactified theory, so the type IIB supergravity in the $AdS_5\times T^{1,1}$ background should lead to $\mathcal{N}=2$ $U(1)$ gauged AdS_5 supergravity coupled with $SU(2)\times SU(2)$ Yang-Mills vector multiplets and some Betti multiplets. While in the K-S solution background the type IIB supergravity should yield a gauged AdS_5 supergravity with local symmetry breaking since the deformed $AdS_5\times T^{1,1}$ is less symmetric. The broken symmetries include the $U(1)$ gauge symmetry, $\mathcal{N}=1$ conformal supersymmetry and part of diffeomorphism symmetry of AdS_5 space. Further, the spontaneous breaking of local symmetry triggers the Higgs mechanism. We work out the super-Higgs effect corresponding to the spontaneous breaking of the above local symmetries and show that the Goldstone fields come from the NS-NS- and R-R two-form fields relevant to fractional branes. Finally, we verify that the Goldstone fields, which is a key ingredient in implementing the super-Higgs mechanism, do constitute an $\mathcal{N}=2$ hypermultiplet in AdS_5 space and further we argue that it may lead to an $\mathcal{N}=1$ chiral supermultiplet on the boundary of AdS_5 space. Since the superconformal anomaly of a

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four-dimensional supersymmetric gauge theory is a chiral supermultiplet, this verification coincides with the prediction from gauge/gravity duality.

The presence of fractional $D3$-branes is the common origin for the superconformal anomaly of the supersymmetric gauge theory and the spontaneous breaking of local symmetries in $\mathcal{N}=2$ $U(1)$ gauged AdS_5 supergravity. Therefore, we conclude that a dual correspondence between the superconformal anomaly on the field theory side and the super-Higgs mechanism on the gravity should be established.

Acknowledgments

I would like to thank Professor M. Chaichian and Professor R. Jackiw for their continuous encouragements and support and O. Aharony, A. Buchel, C.T. Chan, M. Grisaru, A. Kobakhidze, C. Núñez and Y. Yang for discussions, communications and conversations. I am especially indebted to Professors I.R. Klebanov, M. Günaydin and Dr. P. Ouyang for discussions in revising the original version. I would also like to thank my colleagues at the string theory group of Taiwan for comments when I gave several seminars on this topic. This work is partially supported by the National Research Council through NCTS of Taiwan under the grant NSC 94-2119-M-002-001.

A. Γ-matrix in ten dimensions

We choose following explicit representation for Γ-matrices Γ^A in ten dimensions [31, 29],

$$\Gamma^A = e^A{}_M \Gamma^M, \quad \Gamma^a = \gamma^a \otimes 1_4 \otimes \sigma^1, \quad \Gamma^m = 1_4 \otimes \tau^m \otimes (-\sigma^2), \tag{A.1}$$

where $M=(\alpha,\xi)$ are Riemannian indices in ten dimensions, $A=(a,m)$ are local Lorentz indices, $a,\alpha = 0,\cdots,4$, $m,\xi = 5,\cdots,9$; γ^a and τ^m are the Dirac matrices in AdS_5 space and $T^{1,1}$, respectively; $e^A{}_M$ are vielbein in ten dimensions,

$$\begin{aligned}
&\left\{\Gamma^A,\Gamma^B\right\} = 2\eta^{AB}, \quad \left\{\Gamma^a,\Gamma^b\right\} = 2\eta^{ab}, \quad \{\tau^m,\tau^n\} = 2\delta^{mn},\\
&\gamma^0 = -\gamma_0 = i\sigma^1 \otimes 1_2 = i\begin{pmatrix} 0 & 1_2 \\ 1_2 & 0 \end{pmatrix}, \quad \gamma^i = \sigma^2 \otimes \sigma^i = \begin{pmatrix} 0 & i\sigma^i \\ -i\sigma^i & 0 \end{pmatrix},\\
&\gamma^4 = \gamma_4 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \sigma^3 \otimes 1_2 = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}, \quad i=1,2,3,\\
&\tau^5 = \tau_5 = i\gamma_0, \quad \tau^6 = \tau_6 = \gamma_1, \quad \tau^7 = \tau_7 = \gamma_2, \quad \tau^8 = \tau_8 = \gamma_3, \quad \tau^9 = \tau_9 = \gamma_4.\\
&G_{MN} = \eta_{AB} e^A{}_M e^B{}_N, \quad \eta_{AB} = \mathrm{diag}(-1,1,\cdots,1)
\end{aligned} \tag{A.2}$$

The above choice on Γ-matrices determine the ten-dimensional γ_5-analogue,

$$\Gamma^{11} = \Gamma_{11} = \Gamma^0\Gamma^1\cdots\Gamma^9 = 1_4 \otimes 1_4 \otimes \sigma_3 = \begin{pmatrix} 1_{16} & 0 \\ 0 & -1_{16} \end{pmatrix}. \tag{A.3}$$

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